AN

INTRODUCTION

TO

NATURAL PHILOSOPHY.

ILLUSTRATED WITH COPPER PLATES.

By W[redacted]M NICHOLSON.

Non enim me cuiquam mancipavi, nullius nomen fero. multum magnorum virorum judicio credo, aliquid et meo vindico Nam illi quoque, non inventa, fed quærenda, nobis reliquerunt SENECA.

THE FOURTH EDITION, WITH IMPROVEMENTS.

IN TWO VOLUMES.

VOL. I.



LONDON.

PRINTED FOR J JOHNSON, NO. 72, ST. PAUL'S CHURCH-YARD.

MDCCXCVI.

TO

SIR JOSEPH BANKS, BART.

P. R. S.

SIR,

GRATITUDE and respect are due from every individual in society, to him who promotes its real interest, by extending the bounds of knowledge. All Europe is acquainted with the exertions you have made for its advancement: permit me to join the voice of nations, by expressing the sense I entertain of them. With this intention, I take the liberty of dedicating the following Treatise to the President of that respectable Body of Men, among whom the

true Philosophy had its origin, and to whom it owes a great part of its improvements.

I am, with much respect,

Sir,

Your most obedient

humble Servant,

London,
Feb. 28th, 1782.

W. NICHOLSON.

PREFACE.

The advantages derived from the ſcience o[illegible]ral Philoſophy, are ſo great and ſo univerſally acknowledged, that an enumeration of them would be unneceſſary, if it did not ſerve to enliven and direct that ſpirit of inquiry which is natural to youthful minds; and to awaken thoſe who from a want of reflection are not inclined to look into the cauſes of things. We are apt to regard objects to which we have long been familiarized, with languor and indifference, and we now behold effects without even the emotion of curioſity, which in leſs enlightened ages would have been thought miraculous.

Man in a rude and ſavage ſtate, with a precarious ſubſiſtence, expoſed to the inclemencies of the ſeaſons, and the fury of wild beaſts, is an object of pity when compared to man enlightened and aſſiſted by Philoſophy: Ignorant of architecture, of agriculture, of commerce, and of all the numerous arts which depend upon the mechanic powers; he exiſts in the deſart, comfortleſs and unſocial, little ſuperior in enjoyment to the lion or the tyger, but much their inferior in ſtrength and ſafety. If it be true that man ever exiſted in this ſtate, it could not have laſted long; the exertion of his mental ſtrength muſt

hav[illegible] rise to the arts. Aided by these, the wilderness becomes a garden embellished with temples, palaces, and populous cities; and he beholds himself removed to an immense distance from the animals, to which in his original ignorance he seem[illegible]

[illegible] sciences bestow that leisure and independence which have enabled superior minds to form laws, and to establish the rights of mankind by mutual compact between the powerful and the weak. By this leisure it is that ingenious and speculative men have collected masses of knowledge, which induce us to regard the powers of the human mind with astonishment. Hence we possess the admirable science of Astronomy. A science founded on the most accurate and long continued observations, and systemised by the purest mathematical reasoning; but at the same time so remote from vulgar apprehension, that its daily and important uses and predictions are hardly sufficient to prevent its being regarded by the ignorant, as a chimera!

The other departments of Natural Philosophy, are not less replete with wonders. How great would have been the surprise of the antients, could they have foreknown the effects which are produced by the reflection and refraction of light! By a skilful management of these properties, telescopes and various optical instruments are constructed: objects too remote to be perceived by the naked eye, are enlarged and rendered visible: the satellites of Jupiter

Jupiter and Saturn, the mountains [illegible] in the moon, and the changes which take place in the ſun's diſc, are thus diſcovered, and afford matter for admiration and enquiry.

Neither is this delightful ſcience of optics confined to the contemplation of diſtant objects. Minute animals, the veſſels of plants, and, in ſhort, a new world in miniature, is diſcloſed to our view by the microſcope, and an inexhauſtible fund of rational entertainment and knowledge is brought within the ſphere of our ſenſes.

Every one is acquainted with the benefits derived from the ſcience of Hydroſtatics, to which we are indebted for many uſeful inventions. Among theſe are wind and water mills, pumps, fire engines, ſteam engines, &c. &c.

Chemiſtry is productive of great and ſingular advantages to ſociety. Metallurgy in its utmoſt extent, the arts of making glaſs and pottery, of dying, and many others, together with a very conſiderable part of the Materia Medica, are dependant on this branch of Philoſophy. The vaſt importance of Metallurgy, may be rendered obvious from the ſingle conſideration of the many uſes to which iron is applied. Without this metal we ſhould be almoſt totally incapable of making any utenſil or inſtrument. It is difficult to recollect any production of art in the formation of which iron is not made uſe of, and the very exiſtence of naval commerce depends on its magnetical property.

Philoſophy is not therefore a dry ſtudy, but a purſuit of the higheſt utility and entertainment. Thoſe who cultivate the ſciences know that they naturally produce a ſincere and diſintereſted love of truth. An enlarged view of things deſtroys the effects of prejudice, inſpires the propereſt ideas of the great original cauſe, and promotes a deteſtation of every thing that is mean or baſe. And if there be a pleaſure in attending to objects which fill the mind by their immenſity, and delight the imagination by the continual diſcovery of new and ſublime analogies, it is not to be wondered that Philoſophers purſue their ſtudies with a degree of attention and ardor, which is not found in any other ſet of men.

The order of arrangement in the preſent work, is ſuch as was ſuggeſted by the ſubjects themſelves. After a curſory enumeration of the general properties of matter, motion is principally attended to, being that affection of matter by which all changes are brought about. Mechanics and aſtronomy naturally follow, and are ſucceeded by an elucidation of the properties and motion of light. The more complex motions of fluids, and the atmoſpheric phenomena are next conſidered. Thus far it will be obſerved, that the work treats of ſuch general effects as ariſe from the motions of bodies, without any particular reſpect to thoſe ſpecific properties which diſtinguiſh them into various claſſes. The remaining part of the treatiſe is employed upon theſe ſpecific properties: a long ſection upon chemiſtry is given for the

purpoſe

purpose of explaining them as far as they are as present known, and are capable of being understood by mere reading. Magnetism and electricity occupy the concluding sections. Upon the whole, therefore, it will be seen that the most scientific and best established parts of Natural Philosophy are first treated of, and are followed in succession by others which are less understood.

This treatise being intended to give a clear account of the present state of Natural Philosophy, to such as possess very little mathematical knowledge; care has been taken to select such facts and experiments as tend to establish elementary truths. The varieties of experiments of the same kind are not therefore numerous; but it is hoped that the advantage of a greater number of general principles, is by that means obtained. Philosophical instruments likewise, are not minutely described. References to the parts of drawings are not often read or understood: for which reason it was thought better to explain their general construction, and leave the minutiæ to ocular inspection. The grand object throughout has been to relieve the memory, and assist the understanding, by conciseness and illustrative arrangement.

Those prolix disquisitions, which render the commentator less intelligible than the author commented upon, are thus avoided; neither has the affectation of familiarity, which is usually attended with a lax,

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and unphilosophical explanation of one event by another equally obscure, been indulged. On the contrary, the author has every where endeavoured to preserve that solidity of argument, and precision of expression, which distinguish the works of the best Philosophers. And, notwithstanding the nature of the undertaking unavoidably required a deviation from those elegant and general principles which are obtained by strict mathematical reasoning; yet it is presumed that the student will find nothing in this treatise, which he will be under the necessity of unlearning, when he attempts the perusal of those books to which this is offered as an introduction.

The attentive examination of other books, to which the writer of this performance has had recourse, has shewn him, that even the works of those great men, who deserve and possess the highest reputation, are not free from errors of importance. The present occasion does not require the disagreeable task of pointing them out; but this very consideration will not permit him to hope that his diligence has entirely excluded mistakes. However he has little to fear on that account, being sensible that those who are the best able to discover them, will at the same time be the readiest to exercise that candor which every writer has need of.

The liberty which has been taken in altering the words of other authors, and adapting them to the purpose of this work, would have prevented the use

of

of formal quotations if they had been supposed necessary; and as the present intention is not at all historical, the names of authors have been avoided as much as was consistent with the wish of the writer, to evade the suspicion of plagiarism. If plagiarism can be imputed to the author of an epitome of science, this acknowledgment must be allowed to obviate the charge.

In the printing every thing which could be imagined of service to the book, as a manual of philosophy, has been done. A varying title at the head of each page, and copious indexes, are annexed. From these the reader will see that scarcely any fact of importance has been omitted.

The learner who may be induced to fix his chemical reading in his memory by recurring to experiment, which may be done with very little expence, is cautioned to beware of the danger with which it is sometimes attended. The solution, evaporation, and calcination of uninflammable matters, may be performed in the common apartments of a dwelling house; but the distillation of corrosive or inflammable substances, ought not to be attempted but in a place prepared for the purpose. The bursting of a retort containing any concentrated fuming acid, must be very destructive to furniture, as well as prejudicial to health; and ardent spirits, resins, and the like, would endanger the house if a similar accident were to happen. It is impossible to give advice

advice against the many casualties to which chemical experiments are liable. One general maxim is always to endeavour from analogy to foresee the consequence, or probable result, of the intended process, and when that cannot be done, to observe the phenomena, and proceed with caution.

ADVER-

ADVERTISEMENT

TO THE

SECOND EDITION.

The improvements in this ſecond edition are very conſiderable; as well in conſequence of the rapid progreſs of the diſcoveries made by Philoſophers in all parts of the civilized world, as of the careful reviſion the whole work has undergone. The additions are equivalent to a third volume, though by an alteration of the type, and page, this edition has been prevented from exceeding the former in bulk or price. Many additional figures have been inſerted in the plates, and thoſe numbered vii. and viii. are entirely new. Marginal references are likewiſe annexed, which it is hoped will be found eminently uſeful. In theſe the figures denote the pages, and the letters the paragraphs of the pages, where the proofs or illuſtrations referred to are to be found: and in the ſecond volume the numeral letter I. is

is prefixed whenever the first volume is referred to.

London,
Nov. 1st, 1786.

Of the present fourth edition I have only to observe, that the whole has been carefully revised, and the recent discoveries added in their proper places.

London,
January 8th, 1796.

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AN

INTRODUCTION

TO

NATURAL PHILOSPHY.

OF THE DEGREES OR KINDS OF KNOWLEDGE; AND THE RULES OF PHILOSOPHIZING.

THE impreſſions made on the organs of ſenſe by external objects produce ideas in the mind. We are continually employed in amaſſing a ſtock of general truths reſpecting them, which is called knowledge.

An intelligent being, whoſe powers are limited, muſt of neceſſity be unequal to the performance of many things. If any aſſertion be made reſpecting two or more ideas, it will imply either truth or falſehood; but the caſes in which we can with certainty diſcover the one or the other are very few, in compariſon with thoſe that are placed beyond our power of inveſtigation. Yet, as a deciſion is almoſt always required for the regulation of our conduct in the affairs of life, the greater part of our knowledge becomes founded on probability, inſtead of eſtabliſhed truth. The means of acquiring knowledge may therefore be ſaid to be of three

kinds. Certainties are obtained either by intuition or demonſtration; probabilities are obtained by analogy.

There are ſome ideas whoſe mutual relation in certain reſpects is ſo evident, that nothing more is required to obtain the knowledge of it, than to apply them to each other. For example; if a given body be divided into parts, and the mutual relation between the whole body and one of its parts, with reſpect to magnitude, be demanded, the mind immediately conceives, with the cleareſt and moſt abſolute certainty, that the whole body is greater than its part. If the particular body or magnitude in contemplation be abſtracted, or left out, the propoſition becomes general in this form, viz. every magnitude is greater than any part of the ſame. This kind of knowledge is called intuitive, and the general propoſitions are termed Axioms.

When it is required to determine the mutual relation of two ideas, whoſe agreement or diſagreement cannot be intuitively perceived, the truth may often be obtained by the interpoſition of a chain of axioms. This method of exhibiting the truth is termed demonſtration, and ſeems to be applicable only to our ideas of the quantities and poſitions of magnitudes. For this reaſon, it will be difficult to give an example without having recourſe to the mathematicks. The following will, however, be eaſily underſtood.

Fig. 1. Let the two circles in the figure be ſuppoſed equal, and the circumference of each to paſs

through

through the center of the other. Imagine the centers to be joined by the right line A B, and the lines C A, C B, to be drawn from one of the points where the circumferences interfect each other, to the centers refpectively. Then, I fay, the lines A B, B C, C A, will be equal each to each.

The demonftration of this truth is as follows:

The word circle fignifies a plain figure, contained under one line, called the circumference, to which all right lines drawn to a certain point within the figure, called the center, are equal. As foon therefore as it is underftood that the figure A C D is a circle, and that the lines A B, C B, are right lines drawn from its center to its circumference, it is acknowledged intuitively, and without further argument, that thofe lines are equal.

The fame reafon in the circle E C B evinces, that the lines A B, A C, are equal.

The lines A C, C B, being thus proved to be each equal to the line A B, are likewife equal to each other. For it is an intuitive truth or axiom, that things equal to one and the fame thing are equal to each other.

The want of axioms, and the labour of demonftration, are not the only impediments to the acquifition of knowledge. Knowledge is converfant with ideas only: it can therefore be faid to poffefs reality with refpect to eternal objects, fo far only as thofe ideas may be taken or fubftituted for the things they reprefent; but it is impoffible to determine how far this may be done with propriety, even if it can be

done at all. In referring from ideas to things we are liable to error, not only because the compound idea of a being consists of an assemblage of its properties, which may be incomplete and inadequate, but likewise because those ideas may even be quite different from any thing existing in the being itself, as may be instanced in the ideas of colour, sound, pain, &c. The great perspicuity and certainty of mathematical knowledge arises from the simplicity of the ideas employed, and their not depending on any external being: for, as this science treats only of ideas, it is of no consequence to its truths, whether geometrical figures ever had an existence; it being sufficient that their existence is possible.

The greater number of our ideas being too complex and imperfect to admit of intuitive conclusions or axioms, it is evident, that in general we must be contented with less proof than demonstration. Instead therefore of endeavouring to obtain axioms by comparing ideas, we observe events, and from the contemplation of what has happened, we form a presumption of what will again come to pass. Observation has shewn us, that a certain event is always followed by another determinate event; we suppose a relation to subsist between them; we imagine this relation to be necessary; we distinguish the prior event by the name of Cause, and the latter we call the Effect. This kind of knowledge, which is not founded on reasoning, but on experience alone, may be termed Analogical, and is much less perfect than what is obtained by intuition or demonstration.

monſtration. That a ſtone will deſcend to the earth, is an analogical propoſition. It cannot be demonſtrated: but, from the conſideration of a vaſt number of events of the ſame nature, a degree of probability ariſes, which commands our aſſent. It is clear, that analogical propoſitions are no more than ſtrong probabilities, from the remarkable circumſtance, that their converſe does not imply an abſurdity. To deny an intuitive or demonſtrative truth, is to aſſert an impoſſibility; but to deny an analogical truth, is only to aſſert an improbability. The underſtanding revolts at the affirmation, that a part is greater than the whole body; but we ſee no impoſſibility in the aſſertion, that a ſtone, at ſome time or place, has remained in the air without a tendency to deſcend; this ſuppoſition being highly improbable, but nothing more. In fact, demonſtration is a collection of truths or axioms; analogy is a collection of probabilities. Simple probabilities are to analogy what axioms are to demonſtration. Now, there is no compariſon in point of certainty between axioms; all being equally true; but probabilities differ exceedingly in their degree of credibility.

Natural Philoſophy, ſtrictly ſpeaking, admits of no other proofs than thoſe of analogy. To give ſtability to this ſcience, it is neceſſary to admit no probabilities as firſt principles of analogy, but thoſe which poſſeſs the ſtrongeſt and moſt incontrovertible reſemblance to truth. For this purpoſe, the following rules are adopted:

Rules of Philoſophizing.

I.

No more cauſes of natural things ought to be admitted than are real, and ſufficient to explain the phenomena.

II.

And therefore effects of the ſame kind are refered to the ſame cauſes.

III.

Thoſe qualities, whoſe virtue can neither be increaſed nor diminiſhed, and which are found in all bodies with which experiments can be made, ought to be admitted as qualities of all bodies in general.

BOOK I.

SECT. I.

Of Matter in the Abſtract.

CHAP. I.

OF MATTER AND ITS PROPERTIES.

Matter is known to us only by its properties. A

The properties common to all matter are exten- B
ſion, impenetrability, inertia or reſiſtance, attraction, motion, and reſt; all which, except the two laſt, which cannot exiſt together, are found in all bodies whatſoever.

It would be, perhaps, a fruitleſs attempt, to enquire whether theſe are the only qualities with which bodies are endued in common. Matter may poſſeſs many others, that our ſenſes are not adapted to obſerve, or which have even eſcaped the notice of
Philoſophers. But it is neceſſary to obſerve, that C
we are totally ignorant of the ſubſtance in which theſe properties are united. The eſſence of matter is unknown to us. We muſt not, therefore, aſſume one or more of theſe properties as compoſing that

eſſence itſelf; for moſt of the errors of the earlier philoſophers have ariſen from this ſource.

D There are other properties, ſometimes called ſpecific, that are not found in all bodies; ſuch as tranſparency, opacity, fluidity, conſiſtence, and the like. But theſe ſeem to relate to the figures or motions of the parts of bodies, and are, therefore, referable to the general properties. There are alſo ſeveral ſpecies of attraction and repulſion, which will be attended to in their proper places.

Here follow definitions of the general properties above mentioned.

E Extenſion, is that affection of matter by which it occupies part of ſpace.

F Impenetrability, is that by which two bodies cannot exiſt in the ſame place at the ſame time.

G Inertia, is that by which a body reſiſts any force impelling it to a change of ſtate, with regard to motion or reſt.

H Attraction, is that by which one body continually tends to approach to, and, if not by external means prevented, does approach to, another body or bodies.

I Motion, is a continual and ſucceſſive change of place. Reſt, is the permanency or remaining of a body in the ſame place.

CHAP.

CHAP. II.

OF EXTENSION AND IMPENETRABILITY.

The idea of extenſion is ſo ſimple, that it cannot be defined. For though in the preceding Chapter it was pointed out as that affection by which matter occupies part of ſpace, yet, there can be little doubt but that the idea of extenſion is itſelf antecedent to that of ſpace, and therefore not properly definable by it. In order to facilitate the conſideration of ſuch truths as relate to extended magnitudes, geometers have, as it were, analyzed extenſion. It is evident that extenſion implies form or figure, and figure muſt be limited. The limit or termination of figure is called a ſurface, or ſuperficies. A ſuperficies is likewiſe limited, and its termination is called a line. And the termination of a line is called a point. Now, though it is clear, that a ſuperficies, a line, or a point, cannot exiſt ſeparate or apart from an extended being, yet, it is certain that the ideas of them may be conſidered diſtinctly, without immediately referring to the other conſequences ariſing from the general idea of extenſion. In this ſenſe mathematicians define a point to be that which has no part, or is altogether indiviſible; a line to be that which is length, without breadth, or is diviſible in one reſpect, namely, of length; a ſurface to be that which has only length and breadth, or is diviſible in two reſpects, namely, of length and breadth; and a ſolid

to

to be that which has length, breadth, and thickneſs, or is diviſible in three reſpects, namely, of length, breadth, and thickneſs.

o No finite or imaginable diviſion of a line can ever produce a point or indiviſible part. A line is therefore diviſible into an infinite number of other lines, or ſimilar parts, and conſequently much more is a ſuperficies, and yet more a ſolid. A mathematical ſolid, that is to ſay, pure extenſion, is diviſible to infinity: and if the elements of matter be of the ſame nature as the aggregates they compoſe, matter is likewiſe infinitely diviſible. A variety of remarkable conſequences follow from theſe plain deductions; of which we ſhall proceed to mention a few inſtances both theoretical and practical. Thus,

p Any quantity of matter, how ſmall ſoever, and any finite ſpace, how great ſoever, being given (as for example, a cube circumſcribed about the orb of Saturn) it is poſſible for the ſmall quantity of matter to be diffuſed throughout all that ſpace, and to fill it, ſo that there ſhall be no pore or interſtice in it, whoſe diameter ſhall exceed a given finite line.

To prove this, ſuppoſe the cube to be divided into ſmall cubes, whoſe ſides ſhall each be equal to half the given line. It will be eaſily ſeen, that the number of ſmall cubes will not be infinite. Imagine the quantity of matter to be then divided into a number of parts equal to that of the ſmall cubes, and a particle to be placed in the center of each cube. The whole ſpace will thus be filled, and

the greateſt diſtance between two adjacent particles, or, in other words, the diameter of any pore or interſtice, will be leſs than the given line.

Hence there may be given a body, whoſe matter, if it could be reduced or compreſſed into a ſpace abſolutely full, that ſpace may be any given part of its former magnitude.

Again; there may be two bodies of equal bulk, and their quantities of matter may be unequal in any proportion; yet the ſum of their pores, or quantity of void ſpace in each of the two bodies, ſhall be nearly in the proportion of equality to each other.

This is not ſo obvious as the former inſtance; but an example will render it clear.

Suppoſe one thouſand cubic inches of gold to contain one cubic inch of matter, or, in other words, when reduced into a ſpace abſolutely full, to be equal to one cubic inch: then one thouſand cubic inches of* water will contain one nineteenth part of an inch of matter when reduced. Conſequently, the void ſpaces in the gold will be nine hundred and ninety-nine cubic inches, and thoſe in the water nine hundred and ninety-nine cubic inches, and eighteen nineteenth parts of an inch; that is, they will be nearly in the ratio of equality.

Yet, the actual diviſibility of matter can probably be carried but to a certain degree. The ultimate particles of bodies, it is moſt likely, are not to be altered by any force in nature. But there are, nevertheleſs, many inſtances which ſhew to

* Gold is nineteen times as heavy as water.

what inconceivably minute parts bodies may be actually divided.

S A grain of leaf-gold will cover fifty square inches, and contains two millions of visible parts; but the gold which covers the silver wire, used in making gold lace, is spread over a surface twelve times as great.

T In making this wire, it is usual to gild a cylindrical bar of silver strongly, and afterwards draw it into wire, by passing it successively through holes of various magnitudes in plates of steel. By this means the surface is prodigiously augmented, notwithstanding which, it still remains gilded so as to preserve an uniform appearance even when examined with the microscope. The quantities of gold and silver, and the dimensions of the wire, are known. With these data it is easy to calculate, and from calculation it is proved, that sixteen ounces of gold, which, if in the form of a cube, would not measure one inch and a quarter in its side, will completely gild a quantity of silver-wire sufficient to circumscribe the whole globe of the earth.

U The animalculæ observed in the milt of a cod-fish are so small, that many thousands of them might stand on the point of a needle.

Supposing the globules of the blood in these animalculæ to be in the same proportion to their bulk as the globules of a man's blood bear to his body, it appears, that the smallest visible grain of sand would contain more of these globules than

10,256

10,256 of the largest mountains in the world would contain grains of sand.

These instances may serve to shew the amazing fineness of the parts of bodies, which are nevertheless still compounded. Gold, when reduced to the thinnest leaf, still retains those properties which arise from the modification of its parts. Microscopic animalculæ are without doubt organized bodies, and the particles of their circulating fluids must be possessed of specific qualities. Even the rays of light are compounded of an almost infinite variety of particles, which, when separated from each other, exhibit the powers of exciting ideas of colours. None of these are the ultimate particles of which all bodies are formed, for they all bear evident marks of composition. How inconceivably small then must those particles be! v

To these ultimate particles alone it is, that impenetrability can be attributed. Penetration takes place in all compounded bodies. Water exists in the pores of wood. Air in the pores of water. Quicksilver in the pores of gold, &c. &c. w

Some philosophers have questioned whether impenetrability be really a property of matter; and it must be confessed, that, notwithstanding this idea is so closely connected in the formation of our compound idea of matter, yet, if we examine from whence the notion is originally obtained, we shall find that our knowledge is much less certain than we may have suspected. x

To make this clearer, we must consider that our notion of impenetrability is derived from the sense y

of

of feeling. We move the hand towards a body, and in ſituations where motion is not generated, it is prevented by that body from going forward; from which we conclude, that the body poſſeſſes a part of ſpace to the excluſion of every other body; that is to ſay, that it is impenetrable.

But, in order to juſtify this concluſion, it is neceſſary that we ſhould be certain that it is the body itſelf, actually occupying ſpace, which reſiſts the preſſure; and of this we cannot be aſſured, ſince we obſerve many inſtances in which bodies afford reſiſtance to other bodies which move in ſpaces at ſome diſtance from the reſiſting body. Thus, the loadſtone, in certain circumſtances, reſiſts the motion of iron which approaches towards it; and there is no doubt but this reſiſtance or repulſion, if exerted on any part of a man, would afford a ſenſation ſimilar to that which ariſes from contact. If the man had not ſight, or ſome other ſenſe to perceive that the reſiſting body was really diſtant, he would, from the ſenſe of touch, conclude that the body was in contact with the part perceiving; and, if any force he could produce were inſufficient to overcome that reſiſtance, he would conclude the body to be impenetrable.

Now, by ſeveral experiments, which we ſhall have occaſion to mention in the courſe of this work, there is the higheſt reaſon to conclude, that all bodies exert a repulſive force on each other, and that the common effects which are attributed to contract and colliſion are produced by this repul-

ſion:

sion: And, if so, why not attribute all effects of the same nature to this cause, which we know exists, instead of supposing an impenetrability that can never be proved?

If the force of repulsion be sufficiently great, it B
may not be in the power of any natural agent to overcome it, and, consequently, all the effects of a real impenetrability will take place, though the substance or matter itself may not be impenetrable, or even extended.

It is not in our power to determine, whether C
impenetrability or extension be essentially necessary to existence. For the extension of the elements of matter seems capable of no other proof than what may be drawn from their impenetrability; and experiment cannot decide, because a finite pressure can only prove the reaction of a finite resistance.

The question therefore is, whether it be more D
probable, that the particles of matter are beings possessed of a finite power of repulsion, which prevents their mutual approach, but does not render mutual penetration or coincidence in the same part of space impossible, on the application of force sufficient to overcome that repulsion; or whether they be impenetrable atoms, which, consequently, must resist such coincidence with an infinite force?

Here we must attend to the facts. If the repul- E
sion continually increased as the distance of the bodies decreased, we might conclude that it was the only cause of the apparent impenetrability of bodies;

bodies; but, as in the loadſtone, there is a certain ſmall diſtance at which repulſion ceaſes, and attraction takes place, ſo in compreſſing bodies together, with a certain degree of preſſure, the diſtance is at length diminiſhed ſufficiently for the bodies to adhere. The phenomena are probably ſimilar; but, at all events, the coheſion of the parts of bodies ſhews a mutual attraction; and it is not eaſy to explain why the parts ſhould not mutually penetrate and coincide, when the repulſion on which their impenetrability was ſuppoſed to depend has ceaſed, and given place to attraction. And on this account the doctrine of impenetrable atoms would ſeem the moſt probable.

This deduction, however, ſuppoſes the impenetrable particles to come into contact, where attraction has taken place. But it is certain they do not. For things in abſolute contact cannot come nearer without penetration, and cold is known to diminiſh the bulk of bodies; or, in other words, to bring their parts nearer each other than before. This contraction is greater, the greater the cold. Without enquiring into the nature of cold, we may therefore preſume that the utmoſt poſſible cold would either bring the impenetrable particles into abſolute contact, or cauſe the body to vaniſh by the mutual penetration of all its elements. Thus, the original queſtion returns to us, and the only remaining argument ſeems to be——If by the firſt rule of philoſophizing (6) we are to admit no more cauſes of natural things than are ſufficient to explain the phenomena,

phenomena, and we know that a ſphere of repulſion exiſts as the proximate cauſe of our ideas of impenetrability and extenſion, why ſhould we add to this an extended atom exiſting in the center of the ſphere of repulſion?

The quantity of matter in the univerſe is much leſs than is generally imagined. This truth may be deduced from what has been ſaid already on this ſubject; but more eſpecially from the properties of tranſparent bodies. Light paſſes through theſe in all directions without the leaſt difficulty. The focus of a burning mirror, which augments the denſity of the ſun's rays upwards of three thouſand times, may be received in the bodies of glaſs or water, without producing any effect; ſo far are the particles of thoſe ſubſtances from impeding the paſſage of light. And the bottom of the ſea has been diſcerned at a greater depth than ſixty feet. It ſeems not improbable that the real matter in a ſmall piece of glaſs may bear a leſs proportion to its bulk than that bulk does to the whole earth. G

To render the poſſibility of this more evident, we may ſuppoſe a body to be ſo conſtructed, as to have as much vacuity as matter; then half the body would be vacuous. And if the particles of which it is compoſed, be conſtructed in the ſame manner; then the vacuity will become three-fourths of the ſpace occupied by the body. Again, let theſe laſt mentioned particles be conſtructed in the ſame manner; the vacuity will then be ſeven-eighths. H

 And

And if the ſeries be carried forward to the tenth order of particles, the vacuity will exceed the matter one thouſand and twenty-three times.

CHAP. III.

OF THE INERTIA, AND MOTION OR REST.

1 It is chiefly from the inertia that we obtain a knowledge of the relative quantities of matter in bodies. The quantities of matter in bodies abſolutely ſimilar in compoſition, are determined by their extenſion; but in diſſimilar bodies the proportion does not hold. Now, in bodies ſimilar in compoſition, we obſerve that the inertia follows the proportion of the extenſion, that is, by reaſon of the ſimilarity, the proportion of the maſs of matter; and from thence, by applying the proportion of the inertia to diſſimilar bodies, we obtain a knowledge of their maſſes. Thus, for example, the quantity of matter in one cubic inch of gold is as 1, in two cubic inches as 2, in three cubic inches as 3, and ſo forth: this we gather from the extenſion, and alſo from the inertia, both which, in this caſe, follow the ſame proportion. But if a cubic inch of copper be added, though the extenſion be augmented as 1, yet the inertia increaſes only as ½; therefore, either the extenſion or the inertia is not the proper meaſure of the maſs; and, as we can more readily admit that the extenſion,

extenſion, or ſpace occupied within the external limits of the body, may, by poroſity in the body, ceaſe to be the meaſure, than that the inertia of the ultimate parts of matter ſhould vary; we conclude, that the quantity of matter is as the quantity of the inertia; though it muſt be allowed that neither poſition is phyſically demonſtrable.

K The inertia of matter being that by which it reſiſts any change of ſtate with regard to motion or reſt, is meaſured by the force which is required to produced a given change; that is to ſay, the force required to give a certain degree of velocity to a body at reſt A, containing ten parts of matter, is five times as great as would produce the ſame effect on a body at reſt B, containing two parts.

L This force in a moving body is called the quantity of motion, or momentum, and is meaſured by the maſs of matter multiplied by the velocity; for the whole motion of a body is the ſum of the motions of all its parts. Therefore, in the laſt mentioned inſtance, the body A moves with five times the force that B moves with, though the velocity is the ſame in both. But if the velocity of B were to be augmented five times, the quantities of motion would then be equal; that of A being expreſſed by 10, multiplied by 1, and that of B being expreſſed by 2, multiplied by 5.

Motion and reſt are diſtinguiſhed into abſolute and relative. M Abſolute motion is the removal of a body from one part of ſpace to another. N Relative motion is a ſucceſſive change of ſituation with

reſpect to another body, though that body may not be at reſt. Thus, a man ſitting in a barge in motion, is relatively at reſt, that is, with reſpect to the parts of the barge: but abſolutely in motion; being removed, with the veſſel, from one part of ſpace to another. On the contrary, the bargeman, who fixes a ſtaff in the ground, and gives motion to the barge by walking along its gunwale, is abſolutely at reſt, for the ſtaff againſt which he leans is fixed; but relatively in motion, ſince, with reſpect to the veſſel, he walks from one end to the other. But if the earth be ſuppoſed in motion, the abſolute motion of the barge and its contents will be compounded of its relative motion, together with the abſolute motion of the earth.

There is another diſtinction in motion, by which it is called apparent or angular, and which depends on an optical fallacy. Thus, to an eye at E, (fig. 2) a body which moves from C to D, or from F to G, will apparently deſcribe the line A B, though the real motions are very different. And if a body move, either directly towards, or directly from the eye, it will be apparently at reſt. It is true, that, from other circumſtances, we have acquired the habit of diſtinguiſhing different motions which are made obliquely to the eye; but where thoſe circumſtances are wanting, as in the heavens, it requires no ſmall degree of attention to diſtinguiſh the real from the apparent motion.

Experience proves, that the three following laws are sufficient to account for all the phenomena of motion.

LAW I.

Every body continues in its state of rest, or of uniform motion in a right line, unless compelled to change that state by forces impressed.

For matter at rest, being endued with no power of moving itself, would remain so for ever, unless impelled by some external cause.

We have also daily proofs, that a body in motion will continue to move uniformly in a right line, unless prevented or diverted by some other agent. The resistance of the air, and the force of gravity, in a short time destroy the motion of all projectiles, which otherwise, by the vis inertiæ, would continue for ever.

LAW II.

All change of motion is in proportion to the force impressed, and is made in the line of direction in which that force is impressed.

For if any force produces motion, a double force will produce a double quantity, and a triple force a triple quantity, whether it be impressed all at once, or by successive impulses. And this motion, since it always has the same direction as the generating force, if the body be already in motion, either increases the same, by conspiring therewith, or diminishes it by opposition, or is added to it obliquely, being compounded with it according to the directions of the two motions.

Law III.

R Action and reaction are always equal and contrary; or, the mutual actions of two bodies are always equal, and in contrary directions.

Thus, if a ſtone be preſſed by the finger, the finger is equally preſſed by the ſtone. If a horſe draws a ſtone, the ſtone draws the horſe equally backwards, for the rope is equally ſtretched towards both. If one body impels another, it will itſelf ſuffer an equal change of motion by the reaction in a contrary direction. If the loadſtone attract iron, it is itſelf equally attracted, and both will meet at reſt, when they come together, which could not be if they did not preſs equally. By this means the changes of motion, though not of velocity, are always equal.

To illuſtrate this yet more. Suppoſe a horſe proceeds with a quantity of motion expreſſed by the number 3, and that it would require a force equal to 2 to move a certain ſtone. The horſe then drawing, proceeds with a force equal to 1, the reaction of the ſtone deſtroying as much force as the action communicates to it.

From theſe laws the following corollaries are eaſily deduced, which may be applied to ſolve all the effects which can be produced by the mechanical powers.

Corollary I.

S A body impelled by two forces acting in the direction of the two ſides of a parallelogram will deſcribe the diagonal in the ſame time, as by the action

action of one of the forces, it would have described one of the sides.

A body at A (fig. 3) would be carried with an uniform motion in a given time to B, by the single force M impressed at A; and by the single force N, impressed at the same place, would be carried from A to C: complete the parallelogram A B D C, and by the combined forces, it will be carried in the same time in the diagonal from A to D. For since the force N acts according to the direction of the line A C, which is parallel to B D; it will, by Law II. in no respect alter the velocity of approaching to B D, which was produced by the other force. Therefore, the body will in the same time arrive at the line B D, whether the force N be impressed or not; and at the end of the time will be found somewhere in the said line B D. By the same manner of arguing, it will at the end of the time be found somewhere in the line C D, which must of consequence be in that place where they intersect each other. Its motion will be in a right line by Law I.

COROLLARY II.

Hence also appears the composition of a direct force A D, (fig. 3.) from any two oblique forces, A B and B D, and on the contrary, the resolution of a direct force, into any two oblique forces, A B and B D.

The laws of motion being obvious deductions from the phenomena around us, are confirmed by

every mechanical effect we see produced. In the consideration of forces, it is very often convenient to regard them as if compounded of two or more forces, as will be shewn in the ensuing section of this Book. It is clear, that any given motion or force A D may be produced by any pair out of an indefinite number of pairs of forces that may be compounded together; for example, A C and A B, or A F and A E, or, generally, by any two forces, whose quantity and direction may constitute a parallelogram, having A D for its diagonal. In this parallelogram are six things, namely, the directions of the two compounding forces, and of the compounded force, and also their respective quantities. And if any four of these be known, the other two may easily be found, by completing the construction of the figure.

COROLLARY III.

U The quantity of motion which is obtained by taking the sum of the motions made in the same direction, or the difference of those made in contrary directions, is not changed by the mutual action of the bodies.

For action and reaction being equal, by Law III. (22, R) will therefore occasion equal changes in the motions, but in contrary directions. Consequently, if the motions are both made in the same direction, whatever is added to the motion of the impelled body must be subtracted from that of the impelling body, and the sum will remain the same.

But if the bodies meet directly, the quantity of motion deſtroyed being equal in each, the difference of the motions made in contrary directions will remain unchanged.

CHAP. IV.

OF ATTRACTION, CONSIDERED CHIEFLY AS A POWER THAT GENERATES MOTION.

THE force which tends to bring bodies to- v
gether, without any hitherto diſcoverable impulſe, is called attraction. Whether the various kinds of attraction that fall under our obſervation may be referable to one common immediate cauſe, cannot yet be determined. The third rule of phi- w
loſophizing (6) authorizes us to reckon it among the properties of matter; and whether we ſhall ever proceed ſo far as to diſcover, that it is ſecondary to ſome other more remote property muſt depend on future reſearches. It ſeems clear, however, x
that it is not deducible from any of the properties we have enumerated, (7, B) and conſequently that all ſuppoſitions reſpecting the circulation of effluvia, are mere words without meaning.

The ſeveral kinds of attraction are, the attraction y
of gravitation, or gravity; the attraction of coheſion; the attraction of combination, or chemical affinity; the attraction of magnetiſm and the attraction of electrified bodies. Whether theſe be modifica-

tions

tions of one ſimple power produced by variations in the figure, denſity, temperature, or other ſecondary qualities of bodies, is entirely unknown. In this place we are to conſider only the attraction of gravitation as a power that generates or produces motion.

Z Gravitation is that force by which bodies fall to each other. The vicinity of the earth which ſtrongly attracts every thing to itſelf, prevents its effects between ſmaller bodies from appearing; but the attraction of the mountain of Schehallien in Scotland, upon the ball of a pendulum, was found by a very accurate ſet of obſervations to be conſiderable.

A This power is found to act on all bodies, at any given place, preciſely according to the quantity of matter in each, which is diſcovered by the vibration of pendulums, thus. Let the two unequal bodies A and E (fig. 4) be ſuſpended by threads of the ſame length, and be let fall at the ſame time from the points, A and F, in the arcs A C and E G, which are at equal diſtances from the two loweſt points D and H. Then the vibrations of each will be performed in equal times, and conſequently the velocities will be equal. Whence the quantity of motion in each, being the product of its maſs of matter multiplied by its velocity (19, L) will be in proportion to the maſs of matter in each. But (21, Q) the force producing motion is in proportion to the quantity of motion produced. Therefore, the force of attraction is in proportion to the quantities of matter in bodies.

This

This likewiſe appears in falling bodies, all which B being let go from equal heights, how different ſoever in weight, arrive at the ground in the ſame time, that is, with quantities of motion in proportion to their reſpective quantities of matter.

The reſiſtance of the air is not here conſidered, for the ſake of perſpicuity, though it very ſenſibly impedes all motions performed in it. A guinea will arrive at the ground in leſs time than a feather; but in the receiver of an air-pump, out of which the air is exhauſted, they both fall in the ſame time.

Gravitation acts on all bodies at all times, and C that equally, whether in motion or at reſt, as is evident from the velocities of falling bodies, which are uniformly accelerated during the whole of their courſe. That a force conſtantly and equally acting, D will produce an uniform acceleration of velocity, is plain, from the following conſiderations. Suppoſe a body A, begins to move, by the impulſe of gravity impreſſed at that inſtant, with a velocity expreſſed by the number 1, the next inſtant another impulſe will generate a velocity equal to the former. It will therefore move with the velocity 2, and at the third inſtant with the velocity 3, &c. for the preceding velocities are not in any reſpect diminiſhed or altered by the ſucceeding impulſes (21, Q). If then the impulſes are equal, and equidiſtant in time, the generated motion will be uniformly accelerated; and the velocity, which in this caſe may be conſidered as the motion, for the maſs of matter does not alter, will be in proportion to the time; that is, if the velo-

city in 5 inſtants be expreſſed by 5, that produced in 10 inſtants will be 10, &c. This holds good, let the number of impulſes in a given time be ever ſo great. But the number muſt here be conſidered as infinite, for the force ceaſes not to act for the leaſt portion of time, and therefore the acceleration continues uniformly through every part of the motion.

E The ſpace deſcribed by an uniformly accelerated motion in a given time, may be conceived to be the ſum of an infinite number of ſpaces produced by a like number of uniformly increaſing velocities. Theſe ſpaces will be as the velocities by which they are deſcribed. Therefore, as the ſum of the numbers expreſſing the velocities in any given time is to the ſum of the numbers expreſſing the velocities in any other given time, ſo is the ſum of the ſpaces, or whole ſpace deſcribed in the firſt given time, to the ſum of the ſpaces, or whole ſpace deſcribed in the other given time. But the ſums of the velocities, for any terms of time taken from the beginning of the motion, are to each other as the ſquares of the times.

F For the times uniformly increaſing from the beginning, may be expreſſed by the natural ſeries of numbers, 1, 2, 3, 4, 5, &c. &c. The velocities, and alſo the correſpondent ſmall ſpaces, may be expreſſed by the ſame ſeries, becauſe they are both in the ſame ratio of the times. The whole time of deſcription, or the ſum of the inſtants, will be denoted by the number of terms, or, which is the ſame in this ſeries, by the laſt term. And the whole ſpace deſcribed, or ſum of the ſpaces, will be denoted by the ſum of the terms.

Now,

Now, all arithmeticians teach, that twice the ſum of ſuch a ſeries is equal to the number of the terms added to unity, or 1, and multiplied by the laſt term. But, becauſe the inſtants contained in the given time have been aſſumed infinitely ſmall, the number of terms in the ſeries will be infinitely great, and conſequently the addition of unity to the number of terms will not cauſe any finite augmentation. Rejecting therefore the addition of unity, the double ſum of the terms, will be equal to the number of terms multiplied by the laſt term, or, in this ſeries, to the laſt term multiplied by itſelf, that is, the ſquare of the laſt term; and becauſe halves are as their wholes, the ſum of the terms will be in the ſame ratio. But the ſum of the terms expreſſes the whole ſpace deſcribed, and the laſt term denotes either the whole time, or the laſt acquired velocity. Whence G
the ſpace deſcribed from the beginning of an uniformly accelerated motion is as the ſquare of the time, or as the ſquare of the laſt acquired velocity. Which was to be proved.

If the acceleration ceaſe at the end of any given H
time, the motion will become uniform with the laſt acquired velocity, and the ſpace deſcribed in an equal term of time will be double that which before was deſcribed from the beginning by the accelerated motion. For the ſpace deſcribed in any ſingle inſtant of the time by the laſt acquired velocity will be expreſſed by the laſt term of the juſt mentioned ſeries: and the whole ſpace deſcribed will be equal to this ſpace multiplied by the number of inſtants. But the time

or number of inftants being equal to the preceding time of acceleration, will be expreffed by the number of terms, or, in this feries, by the laft term. The fpace defcribed will be therefore expreffed by the laft term multiplied by itfelf; that is to fay, it will be the double of the fpace defcribed by the accelerated motion in an equal time.

I When it is faid that a number denotes any magnitude, it is to be underftood that the number is part of a feries, whofe terms vary in a ratio always correfpondent, or equal to the ratio of the magnitudes denoted; that is to fay, the ratio between any two terms of the feries is always equal to the ratio between the two magnitudes that correfpond to thofe terms. In this fenfe, any magnitudes, however unlike, may reprefent each other. Here follows a geometrical demonftration of the foregoing propofitions, in which lines are made ufe of inftead of numbers.

K In the triangle A B C, fig. 5, let the equal divifions A, 1, 2, 3, &c. on the fide A B, reprefent equal parts of the time of an uniformly accelerated motion. Then the parallel lines, 1 d, 2 e, 3 f, &c. may reprefent the velocities at the feveral inftants, 1, 2, 3, &c. for they are in proportion to the times A 1, A 2, A 3, &c. And in like manner for any other part of the time as A m, the velocity generated will be reprefented by its correfpondent ordinate m n. And the fum of the ordinates correfponding with any part of the time will reprefent the fum of the velocities. But the fum of the ordinates, when taken indefinitely

indefinitely numerous, may be conceived to occupy the area contained between the ordinates of the firſt and laſt inſtants of the time. And theſe areas, when taken from the beginning A, are as the ſquares of the times A 1, A 2, A 3, &c. or of the velocities 1 d, 2 e, 3 f, &c. by the property of ſimilar figures. Therefore the ſums of the velocities, and conſe- L quently the ſpaces deſcribed in any given terms of time taken from the beginning of an uniformly accelerated motion, are to each other as the ſquares of the times, or of the laſt acquired velocities. Hence M it likewiſe appears, that the ſpaces deſcribed in equal ſucceſſive parts of time, are as the areas contained between A and 1 d, 1 d and 2 e, 2 e and 3 f, &c. which areas are to each other as the odd numbers 1, 3, 5, 7, 9, &c. as appears by inſpection from the number of equal and ſimilar ſmall triangles contained in each.

Again, ſuppoſe the motion at the end of the time N A B to become uniform with the laſt acquired velocity B C. Complete the parallelogram B D E C, making B D equal to A B, and the ordinates 1 i, 2 k, 3 l, &c. will denote velocities, and conſequently ſpaces deſcribed. Their ſum will expreſs the whole ſpace deſcribed in a time equal to A B, and will be denoted by the area B D E C. But this area is equal to twice the area A B C. Conſequently the ſpace deſcribed O by an uniform motion with the laſt acquired velocity, during a time equal to that of the acceleration from the beginning, will be double the ſpace deſcribed by the accelerated motion.

It

P It may readily be apprehended that an uniformly retarded motion is exactly the reverse of a motion uniformly accelerated. For suppose a constant force acting against a body in motion; as for example, gravity acting against the motion of a body projected directly upwards, it will destroy an equal part of the initial velocity in an equal particle of time. Now, if these equal deductions be called unities, and be successively taken from any number whatsoever, till the last remainder be nothing, it is evident that the series of remainders will be the natural numbers 1, 2, 3, &c. in a reversed order, and every thing that was proved of the times, spaces, and velocities (28, F. 30, K), or of the parts of the triangle ABC, fig. 5, will be true, mutatis mutandis, that is to say,

Q In the same body in motion, and retarded by a constant and equally acting force, the spaces described in coming to rest, are as the squares of the
R initial velocities (29, G. 31, L), or as, the squares
S of the times during which they are described: and are equal to half the spaces, which in an equal time would have been described by their respective initial velocities uniformly continued.

CHAP.

CHAP. V.

OF THE PRODUCTION, COMMUNICATION, OR DESTRUCTION OF MOTION.

Motion is produced, deſtroyed, or changed T
in a body, either by the impulſe, colliſion or ſtroke of another body in motion, or by the force of attraction. Repulſion being immediately the contrary to attrac- U
tion, and not being perhaps ſufficiently general and univerſal (6) to be admitted as a common property of bodies, need not be here conſidered.

We do not know whether the diſtinction be- V
tween impulſe and attraction be real, and exiſting in the nature of things, or only relative to the imperfect ſtate of our knowledge. The moſt obſervable W
difference is, that impulſe is a force which acts from place to place, or, in other words, cannot be without motion: but attraction can exert itſelf even though no motion is produced.

To exemplify this, ſuppoſe two bodies to meet X
directly with equal quantities of motioh; the effect of the ſtroke will be, that the whole motion will be deſtroyed, and the bodies will remain together. The forces will likewiſe be deſtroyed, and the bodies may be moved apart; each with the ſame facility as if the other did not exiſt. In this caſe, we have ſuppoſed no attraction to be exerted by them on each other. But let it now be ſuppoſed, that their motion, inſtead of being uniform, and the conſequence of their inertia, is produced by a mutual attraction. They come together, and the motion is deſtroyed as be-

fore. But the force of attraction, by which they were originally put in motion, remains, and is exerted in pressing them against each other.

Y The smallest finite impulse can overcome the greatest finite pressure. For, let any pressure be supposed to produce acceleration, and the body, when in motion, will have more force than when it was merely pressed. In its acceleration from rest, it must pass through every possible velocity less than the velocity last acquired. Let the impelling body have a momentum expressed by the product of its mass into its velocity (19, L). Whatever product this may be, it is possible to assume a period of the acceleration of the body pressed, in which its velocity shall be so small, as that its product into its mass shall be still less. And it has already been said, that the force of mere pressure is yet less than this. Consequently, it is less than that of the impulse.

Z From this it is inferred, if two bodies, perfectly hard, or unyielding, were to strike each other with any velocity, that they would be broken to pieces, provided the cohesion of their parts were less than infinite.
A But if the cohesion were infinite, it is presumed that the communication or destruction of motion would
B be instantaneous. However, there are no such bodies found in nature, and very considerable difficulties arise in the abstract reasoning concerning them.

C There appears to be the same relation between pressure and momentum as between a line and a surface.

ſurface. In ſome reſpects, the firſt may be ſaid to generate the latter. Both are capable of increaſe or diminution, and yet no increaſe or diminution of the one can produce the other.

That preſſure, which gravity cauſes bodies to exert againſt any obſtacle interpoſed between them and the earth, is called Weight. We have already taken notice of its effect when free to produce an uniform acceleration in falling bodies (26, A B C). If its power could be increaſed or diminiſhed, it would proportionally increaſe or diminiſh the momentaneous velocities and ſpaces, (28, E) and conſequently the whole ſpace paſſed through in a given time; that is to ſay, conſtant forces are as the ſpaces paſſed through by acceleration in a given time, or as the laſt acquired velocities. D E

When the effect of any retarding force is conſidered (31, P), the force will, in a given time, be as the whole ſpace deſcribed during the retardation, till the motion is deſtroyed, or as the initial velocity. F

Let two accelerating forces be to each other in any ratio, the laſt acquired velocities will (35, E) be in the ſame ratio. Imagine the leſs motion to be continued till its laſt acquired velocity becomes equal to that of the other, and the whole time (27, D) will then be to the former time as the greater velocity to the leſs, or inverſely as the forces; that is, the times required to produce equal velocities are inverſely as the accelerating forces. But the ſpaces deſcribed in equal times are as the forces (35, E). Whence the ſpaces deſcribed in any other G

times will be as the forces and the squares of the times (29, G) jointly. But when equal velocities are produced, the squares of the times will be inversely as the squares of the forces (35, O). Therefore the spaces in this case will be as the forces directly, and the squares of the forces inversely, or

H inversely as the forces; that is, the spaces passed through in producing equal final velocities are inversely as the accelerating forces.

I When the effect of a retarding force is considered (31, P) the spaces passed through in destroying motion are inversely as the retarding forces, when the initial velocities are equal.

K If a body be acted upon by a constant and invariable force, and its mass be either increased or diminished, without altering the force, the effect will be the same with respect to acceleration, or retardation, as if the force, without changing the body, were diminished or increased in the inverted ratio of the mass. For the force being supposed invariable, will always produce or destroy the same quantity of motion (21, Q) in a given time. This quantity will be measured by the product of the mass into the velocity (19, L). And that this product may continue unaltered, it is necessary that the velocity should diminish in the same proportion as the mass is increased, and the contrary.

L A body impinging with different velocities on tallow, clay, timber, and some other substances, penetrates to depths in the same substance which are as the squares of the initial velocities. Whence it follows,

follows, (32, Q) that these substances oppose a constant and invariable force of retardation against the motions of given bodies.

Suppose a body to impinge on an uniformly resisting substance, if the initial velocity vary only, it will penetrate to depths which are as the squares of the velocities (32, Q). But if the mass (not magnitude) vary only, the consequence will be the same as if the retarding force had varied in the inverted ratio of the mass (36, K). And the depths or spaces will be inversely as the retarding forces (36, I) or directly as the masses: consequently, if both the mass and velocity vary, the depths will be in the compound ratio of the masses and the squares of the velocities. M

The dispute concerning the measure of forces, which divided the philosophical world for considerably more than half a century, was founded on a partial consideration of the effects of collision. The question agitated was, whether the forces of bodies in motion ought to be measured by the mass of matter multiplied by the velocity, or by the mass multiplied by the square of the velocity. The former affirmation was called the old opinion, and the latter the new opinion. N O

Neither of these opinions are sufficiently general to apply to every case of motion, neither are they repugnant to each other, as the contenders for each insisted. The chief argument urged by the maintainers of the new opinion was, that spheres of equal magnitude, but of different weights, being let fall into tallow, from heights that were inversely as the weights, P Q

weights, made pits of equal depths in the same. Now, said the disputants, equal causes are those which produce equal effects; the forces of these bodies at their impact on the tallow must be equal, as being the causes of equal effects, namely, the pits in the same substance. But the squares of the velocities of the impacts are (29, 0) as the heights from whence the bodies fall, or in this case inversely as the weights of the bodies. Therefore the product of each weight or mass into the square of its velocity is equal to the product of the other weight into the square of its velocity, when the pits, or, as it is affirmed, the forces, are equal.

R All this is true, when it is considered as a mere explanation of the meaning of the word force, which, if understood and applied in this sense, will
S not be productive of error. But when the above is intended to serve as a proof that the action of a body in motion cannot be measured by the mass multiplied into its velocity, it becomes necessary to observe, that the pit in the tallow being equal to another pit, does not prove that they were made by
T equal powers or forces. For powers cannot be said to be equal, unless they produce equal effects in equal times; it being easy to imagine, that a weaker power continued for a longer time may produce an effect equal to that of a power of greater intensity,
U though of less duration. And it is evident, that these pits are not described in equal times; for they are equal to half the spaces which would have been described with their respective initial velocities uni-

formly

formly continued (32, s) in the reſpective times of deſcription. But the initial velocities would deſcribe ſuch equal ſpaces in times which are inverſely as the velocities themſelves. And it has been already ſeen, that the reſult of the preſent experiment is an eaſy conſequence of the properties of retarded motion, conſidered jointly with that definition which affirms the force or quantity of motion in a body to be as the product of the maſs into the velocity (37, M). v

The conſideration that moving bodies penetrate obſtacles to depths which are as the maſs of the body multiplied by the ſquare of its velocity, is of great uſe in almoſt every circumſtance of this kind. It follows from hence, that the depths to which a body of given magnitude will penetrate into any ſubſtance, may be varied to infinity, without changing the quantity of motion. For the depths will always be greateſt when the velocity is greateſt (37, M); and the quantity of motion, or product of the maſs into the velocity, will not be changed, if the maſs be diminiſhed proportionally while the velocity is augmented, and the contrary. w x

Thus it is ſhewn, that a ſmall hammer, having the ſame ſtriking ſurface and quantity of motion, will do more work at a blow than a large one. The driving of nails, or of piles into the earth, follows nearly the ſame law, though in the inſtance of the engine for driving piles by the fall of a weight, nothing (37, Q) would be gained by leſ- y

ſening the weight and raiſing it higher: becauſe,
from the property of the mechanical engines, the
heights to which weights can be raiſed in a given
time are inverſely as their weights.

z When a body in motion ſtrikes another at reſt,
it does not communicate the utmoſt quantity of
motion to this laſt, until its whole action has been
exerted, which, if it penetrates, is not until it has
either penetrated to its utmoſt depth, or paſſed
A through. Hence a projectile may paſs through an
obſtacle without communicating any conſiderable
quantity of its motion, provided the obſtacle be
conſiderably leſs thick than the depth to which the
projectile muſt have penetrated by its whole effort.
B This is exemplified in a piſtol-bullet ſhot againſt a
door ſet open, through which it paſſes without
communicating motion enough to overcome the
friction of the hinges: whereas a large piece of lead,
having the ſame momentum, inſtead of penetrating,
carries the door before it, even though the ſtriking
part be a prominence no larger than the bullet in
the former caſe.

C The different effects of motion, according as the
velocity is greater or leſs, is ſhewn likewiſe in the
vulgar experiment of breaking a ſtick, whoſe ends
reſt on two wine glaſſes. Fig. 6. Let the two
wine-glaſſes A, C, be filled, and let the ſtick A C be
placed with its ends reſting on the two inner edges,
as in the figure. Then, if a quick blow be ſtruck
downwards, on the middle part B, the ſtick will be
broken aſunder, without diſturbing either the glaſſes

or their contents. In this experiment the two pieces A B, B C, are made to revolve on the points E and D, so that the points A and C rise up instead of pressing the edges of the glasses. On this account, if the blow at B be struck underneath instead of above the stick, the glasses will be broken.

That power or property by which a body recovers its figure, after it has been changed by any external action, is called elasticity. D

If a body strike another at rest, and one or both of them yield inwards, or change their figure, the latter body will gradually, during the time of change, pass through every possible velocity from rest or nothing, to that which is expressed by dividing the quantity of motion in the striking body by the sum of the two bodies (19, L 24, U); at which last instant both bodies will have the same velocity, and will proceed uniformly together, provided neither be elastic. E

But if both bodies be perfectly elastic, they will yield inwards, and the gradual change of velocity will obtain as before, till the instant of the utmost yielding, at which time both will have the same velocity. But the elasticity being supposed perfect, both bodies will recover their figure with a force equal to that by which it was changed. The action of the elastic force being contrary to that of the former stroke, will cause the two bodies to separate with the same relative velocity as they before came together. That is to say, the striking body will lose twice as much motion as it would have F

lost

loſt by a ſimilar colliſion without elaſticity, and the body ſtruck will in like manner gain twice as much. Conſequently the ſtriking body, after the colliſion, will either proceed forward, remain at reſt, or be reflected back, according as its maſs is greater, equal to, or leſs than that of the body ſtruck.

The experimental methods of illuſtrating the theorems relating to motion have been made, for the moſt part, on pendulous bodies, or on bodies let fall, either along inclined planes, or in the open air. Mr. John Whitehurſt, F.R.S. has contrived an inſtrument for meaſuring the time a body employs in falling through a given ſpace. Its chief advantage conſiſts in meaſuring, to an accuracy, conſiderably beyond the reach of the ſenſes. The ſenſe can with difficulty divide a ſecond into twelve parts ſo as to reckon them: but this inſtrument divides it into one hundred. A hand or index, connected with wheel work, is made to revolve uniformly in a circle divided into one hundred equal parts, each revolution being performed in one ſecond. The regulator of the motion is a fly, whoſe leaves may be ſet ſo as to diſplace a greater or leſs quantity of air, accordingly as the inſtrument is required to go ſlower or faſter; or the ſame adjuſtment may be more accurately obtained by altering the weight that gives motion to the wheels. By the conſtruction of the inſtrument the body is let go at the beginning of a revolution, and at the end of its fall it ſtrikes another part of the mechaniſm, that inſtantly ſtops the hand, or index.

Mr.

Mr. G. Atwood, F. R. S. has invented an inſtrument for meaſuring the ſpaces paſſed through in a given time by a body in motion, whether that motion be accelerated, retarded, or uniform. It conſiſts of two cylindrical boxes, ſuſpended at the ends of a fine ſilk line that paſſes over a wheel or pulley. The axis of the pulley reſts on the circumferences of four other wheels, ſo that the effect of friction is ſcarcely ſenſible. If the two boxes be equally loaded, the weight of the one will counterpoiſe or deſtroy the effect of the gravity of the other. The two boxes with their contents, together with part of the pulley, may, in this caſe, be conſidered as compoſing a maſs void of gravity. Let any weight be added to one of the boxes, and as this weight is at liberty to move, it will be uniformly accelerated by the conſtant action of gravity (27, D): but by the conſtruction of the inſtrument, it muſt give an equal velocity to the whole maſs before mentioned, which, therefore, may be ſaid to be an increaſe of its maſs, while the moving force remains conſtant. The ſpaces paſſed through in a given time by the body will, therefore, be to the ſpaces it would have paſſed alone by the ſame action of gravity, (36, K. 35, E) as its own maſs or weight is to the whole maſs in motion. By this happy contrivance, the ſpaces paſſed through by acceleration or otherwiſe, in a given time, are rendered ſhort, and eaſy to be obſerved with preciſion. For this intention there is a chronometer beating ſeconds added to the inſtrument, and a graduated rule near one of the boxes, with a K

moveable

moveable ſtage, to limit or terminate the motion*.

CHAP. VI.

OF THE ATTRACTION OF COHESION, AND OF SPECIFIC ATTRACTIONS.

L THE attraction of coheſion is that force by which bodies or their particles adhere to each other.

M It is demonſtrated, that if the forces by which the particles of bodies tend towards each other decreaſe in the proportion of the ſquares of their diſtances, the attractive force of two ſpheres compoſed of ſuch particles, will be governed by the ſame law; relation being had to the diſtances of their centers: and conſequently, it will not be ſenſibly greater when they are in contact, than when
N they are at a ſmall diſtance from each other. But if the firſt mentioned forces decreaſe in the proportion of the cubes of their diſtances, or in any greater proportion, the latter will decreaſe after a much higher rate, and the bodies, when in con-

* This inſtrument, and its various uſes, are deſcribed at large by the inventor, in " A Treatiſe on the rectilinear motion and " rotation of bodies."

tact, will attract each other very much more forcibly than when feparated at the leaft diftance from each other*. Y

The firft of thefe attractions is gravity, as is evinced by its action on the planetary bodies, and the latter appears to be the attraction of cohefion, for its force is vaftly lefs at the leaft diftance, than at the place of contact. O P

In confequence of this law, feveral deductions are made, which are found to agree with the phenomena of this latter kind of attraction, as,

Thofe particles which are poffeffed of large furfaces of contact, adhere moft ftrongly together, and form bodies which are called hard. Q

Thofe particles which touch each other in few points, compofe bodies which are foft or fluid, on account of the fmall force with which their parts adhere together. R

And hence probably the elafticity of fome bodies may be explained; for it feems to depend on the cohefive force which reftores the particles to their firft relative fituation, when by any external impulfe, they have been removed to a very fmall diftance from each other. S

Many difcoveries remain for the induftry of future philofophers to make concerning this very powerful agent in nature. T

By this power the drops of all fluids affume a round form, and polifhed plates of metal adhere U

* Principia I. 76.—I. 86.

together with a prodigious force. This laſt is exem-
v plified by paring a ſmall part from each of two leaden bullets, and preſſing the ſurfaces together; in which caſe, with a ſurface of contact not exceeding the twentieth part of a ſquare inch, it will frequently require the force of 100lb. to ſeparate them.

w By this power alſo it is, that liquids riſe into the ſubſtance of bread, ſponge, and other porous bodies; and are ſuſtained in open capillary tubes a conſiderable height above the level. This height is in the reciprocal proportion of the diameters of the tubes.

x Two plain glaſs plates, fig. 7. touching each other at the line A B, and ſeparated at C D, by a ſmall obſtacle K, being placed in the veſſel of water E F G H, the water riſes between them to the line D I A, which is an hyperbola.

Let two plain glaſs plates A B C D, fig. 8, be lightly moiſtened with oil of oranges, and placed one upon the other, ſo as to touch at the line A B, being kept ſeparate at C D, by the ſmall obſtacle L interpoſed. In this ſituation let them be placed in the horizontal box, E F G H, the part C D reſting on its bottom, and the other part towards A B, reſting on the upper end of the perpendicular ſcrew I K, which is fixed in the box for the purpoſe of raiſing the plates to any deſired angle of elevation. Then a drop of the above mentioned oil being applied in the opening C D, will be attracted by the two plates, and will proceed with an accelerated mo-

tion

tion towards A B, if the plates are kept in an horizontal poſition. But if the end A B be raiſed by means of the ſcrew, to a conſiderable angle, the drop will remain ſuſpended in its courſe ſomewhere between C D and A B, ſuppoſe at N, and if the elevation be encreaſed, it will return towards C D, its weight overpowering the attraction of the plates.

Now, ſince the weight of the drop continues unaltered, it will not be difficult to find its tendency to return, or that part of its weight which is exerted in the inclined deſcent. For the proportion between that part and the whole, is as the height of the plane to its length, as will hereafter be ſhewn. And ſince the two powers, namely, the attraction by which the drop tends upwards, and that part of its weight which is exerted in the contrary direction, are equal when it remains ſuſpended, the meaſure or quantity of the one will expreſs the meaſure or quantity of the other. By theſe means it is eaſy to determine the attractive force; which is found to increaſe in the reciprocal proportion of the ſquares of the diſtances of the middle of the drop, from the end where the plates are in contact. That is, ſimply in a reciprocal proportion, becauſe the drop enlarges its ſurface as the ſpace becomes narrower; and again, ſimply in a reciprocal proportion, becauſe the attraction increaſes, the nearer the plates approach each other.

This coheſive attraction extends to an extremely ſmall diſtance from bodies, and where its power

terminates, repulſion takes place, of which we ſhall ſubjoin a few inſtances.

A All hard bodies require a conſiderable force to bring them into contact, as appears by compreſſing a convex lens and plane glaſs together, which exhibit different appearances at the very point of ſuppoſed contact, according to the different degrees of compreſſion. This is likewiſe ſhewn from the paſſage of the electric matter through metallic chains. Of which more hereafter.

B When it rains on the ſurface of a veſſel of water, ſmall drops may frequently be ſeen running in all directions, which do not mix with the reſt of the water for ſeveral ſeconds.

C Hence likewiſe it is, that a body ſpecifically heavier than any fluid may be made to ſwim on its ſurface; for, if a quantity of the fluid be diſplaced by the repulſion equal in weight to the ſolid, it will not ſink.

D Dry needles or thin plates of metal ſwim on water, and form cavities of a curve-lined form, extending to a conſiderable diſtance from the body.

E Let A C B, fig. 9. repreſent the ſection of a veſſel of water, on whoſe ſurface A B is laid two circular plates of tinfoil, on each of which is placed a ſmall curtain ring, or ſome ſuch body, to encreaſe its weight, and cauſe it to ſink farther beneath the ſurface. By this means they may form two cavities about one-tenth of an inch deep, and extending half an inch every way from the circumferences of the plates. If they are brought within the diſtance of an inch from each other, they will ruſh together with an accelerated motion.

Things

Things remaining as in the laſt experiment, let F
D and E, fig. 10, be two pieces of wet cork of the ſame dimenſions; the water then, by adhering to their ſides, may form a curve lined protuberance, extending about half an inch from their circumferences, and when they are brought within an inch of each other, they will ruſh together as before.

Theſe appearances are eaſily accounted for, by G
conſidering, that action and reaction are equal. The plate of tinfoil, by its repulſion, acts on the water, and prevents its filling the cavity, and the water by its weight reacts on the plate; but as this reaction is the ſame on all ſides, no motion is produced. But when the two plates approach each other near enough to unite their repulſive actions, the weight of the water between them being diminiſhed by the depreſſion, its reaction is leſs than that which prevails on the oppoſite parts of the circumferences: conſequently they move in the direction of the greater preſſure, that is, towards each other.

In the latter caſe, the reaction is in a contrary H
direction, being oppoſed to an attractive, inſtead of a repulſive force: for the reaction of the water endeavouring on all ſides to return to its level, produces no motion, becauſe its height is the ſame at equal diſtances all round. But when the two attractive powers are, by bringing the bodies nearer, made to act on the ſame water between them, it is raiſed higher above the common ſurface, and conſequently reacts more ſtrongly on that ſide; whence the bodies

rush together, as in the former case. A depression of the surface between the two corks will, by diminishing the quantity of water, occasion them to recede from each other.

I It is not yet decided whether the attraction of cohesion, or the power by which bodies retain the aggregation of their parts, be one and the same with the attraction of combination, or chemical affinity. But as far as experiment has yet extended on this subject, there is reason to believe that they are the effects of different powers.

K When the rational method of philosophizing from observation and experiment was less known and esteemed than it is at present, many objections were made to the admitting attraction as a general cause. Among others, it was said to be a revival of the trifling philosophy of occult causes. But nothing can be more inconsistent and absurd than to compare that philosophy, which deduces general laws from the observation of phenomena, with the compendious method that vanity has invented to disguise or conceal human ignorance, by referring particular facts to occult causes. The laws of motion, the extension, the inertia, the resistance, and the attraction of matter, are deduced in the same manner; and if the causes of these be occult or unknown, it is not that philosophers are unwilling, but because they have not yet been able to discover them, or because some of them are, perhaps too, so simple, as to be referred to any other causes which our senses are adapted

to

to difcern. Nothing, however, can be clearer, than that the exiftence of an agent or caufe may be known, though it may itfelf arife from fome prior caufe of which we are ignorant; and it is enough that attraction really exifts and acts according to eftablifhed laws, to juftify philofophers in admitting it in their explication of natural appearances.

BOOK I.

SECT. II.

Of Bodies in Motion.

CHAP. I.

OF THE MECHANICAL POWERS.

When two heavy bodies or weights are made L
by any contrivance to act againſt each other, ſo
as mutually to prevent each other from being put
into motion by gravity, they are ſaid to be in equi-
librio. The ſame expreſſion is uſed with reſpect to
other forces, which mutually prevent each other
from producing motion.

Any force may be compared with gravity, con- M
ſidered as a ſtandard. Weight is the action of gra-
vity on a given maſs (35, D). Whatever therefore
is proved concerning the weights of bodies will be
true in like circumſtances of other forces.

Weights are ſuppoſed to act in lines of direction N
parallel to each other. In fact, theſe lines are directed
to the center of the earth, but the angle formed
between any two of them within the ſpace occupied
by a mechanical engine is ſo ſmall, that the largeſt

and moſt accurate aſtronomical inſtruments are ſcarcely capable of exhibiting it.

o The ſimpleſt of thoſe inſtruments, by means of which weights or forces are made to act in oppoſition to each other, are uſually termed Mechanical Powers. Their names are, the Lever, the Axis and Wheel, the Pulley or Tackle, the inclined Plane, the Wedge, and the Screw.

p In the theoretical conſideration of theſe ſimple inſtruments, the parts they are compoſed of are imagined or ſuppoſed to poſſeſs no other properties than thoſe which conduce to the purpoſe of their conſtruction. Thus, they are all ſuppoſed to be without weight or inertia, and to move without friction. Many of theſe parts are taken to be mathematical lines, ſome perfectly inflexible, and others perfectly flexible, repreſenting ropes. And theſe ſuppoſitions are allowable, becauſe they imply nothing more than that the reaſonings relate only to abſtract notions or perfect inſtruments; and it is conſequently no more to be urged that there are no perfect inſtruments, than that there are no perfect mathematical figures, at leaſt that ſenſe is able to diſcover or diſtinguiſh. For the difference between theory and practice is in ſome caſes inconſiderable, and may in general be allowed for without much difficulty, from the general principles of mechanics, when once eſtabliſhed.

CHAP.

CHAP. II.

OF THE LEVER.

THE lever is defined to be a moveable and inflexible line, acted upon by three forces, the middle one of which is contrary in direction to the other two. Q

One of these forces is usually produced by the reaction of a fixed body, called the fulcrum. R

Let A C (fig. 11) represent an horizontal lever at rest. At the point B, equidistant from A and C, is placed the fulcrum D, and at the extremities A and C are hung the equal weights E and F. Then the lever will continue at rest, the weights E and F being in equilibrio. For it is evident, that if the line A C be moved on the fulcrum B, its extremities A and C will each be carried with equal velocities in the periphery of the same circle. And because A C is horizontal, the actions of the weights E F will be in direction at right angles to its length: that is to say, they will act in the direction of tangents to the said circle at the points A and C; or they will act in the direction of those particles of the periphery, which may be imagined to coincide with the tangents. Each pressure therefore tends to move the correspondent extremity of the line A C in that very line of direction in which only it can move. Suppose the pressure at C to be removed, and the whole pressure at A will be employed in depressing the point A, or, which is necessarily in this case the S

ſame, in raiſing the point C: on account therefore of the equal velocities of the points A and C, the action of E at the point A will be the ſame as if it were exerted at C in the direction of the tangent C G. But again, ſuppoſe the equal weight F to be reſtored, and the point C will then be acted on by two equal and oppoſite forces, which, deſtroying each other's effect, will not produce motion; conſequently the lever will continue at reſt.

T It is likewiſe evident, that if the radii A B and B C be not in a right line, the equal forces will nevertheleſs be in equilibrio, if they be applied in the directions of the tangents: thus, if B C be bent to the poſition B K, and the force F be there applied in the direction K H, the equilibrium will remain as before.

U If two contrary forces be applied to a lever at unequal diſtances from the fulcrum, they will equiponderate when the forces are to each other in the reciprocal proportion of their diſtances. For,

V Let A C (fig. 12) repreſent a lever, whoſe radius A B is three times as long as B C. At A is ſuſpended the weight E of one pound, and at C is ſuſpended the weight F of three pounds. Then, I ſay, theſe weights will equiponderate. With the radius B A deſcribe the arc A K, interſecting C F at K. Draw the line B K, which may repreſent another arm to the lever; and it will be evidently of no conſequence, whether the thread C F be faſtened at C or K; conceive it therefore to be faſtened at K, and to act on the arm B K. Let A G repreſent the force of E, and K F, being three times as long, will

repreſent

represent the force of F. This force K F may be resolved (23, T) into two others, K I in the direction of B K, and K H in the direction of the tangent, and their quantities are determined by drawing the lines F H and F I, parallel respectively to K I and K H. Now, K I has no effect in moving the arm B K. It is the force K H alone that tends to produce motion towards H. The triangles B C K and K H F are similar, therefore, B K : B C : : K F : K H. But B K : B C, as 3 to 1, whence the force K H is $\frac{1}{3}$ of K F, as is likewise A G by the condition. Consequently H K and A G are equal, and being applied at the end of equal arms A B, B K, will be in equilibrio (56, T), which was to be proved; and the conclusion will be the same, when the weights are to each other in any other ratio, provided the arms of the lever A B and B C be reciprocally in the same proportion.

By the resolution of force it appears, that if two W
contrary forces be applied to a strait lever at distances from the fulcrum in the reciprocal proportion of their quantities, and in directions always parallel to each other; the lever will remain at rest in any position.

For, let the forces, A E, C F (fig. 13) be resolved: X
A E into G E parallel, and G A perpendicular to A C; and C F into H F parallel, and C H perpendicular to A C; and the forces, which tend to produce motion, will in all positions be to each other in the ratio of the forces applied; i. e. A E : C F : : A G : C H, the triangles A G E and C H F being similar.

Many

Y Many curious and uſeful effects may be produced by levers, whoſe arms are bent into an angle; but the limits of this work do not permit us to enlarge upon them.

Z It is evident, that all which has been ſaid concerning the lever is equally true, when the contrary forces are applied on the ſame ſide of the fulcrum.

A On the lever A B, (fig. 14) if the weight E of one pound be applied at A, and the weight F of three pounds at C, ſo that their diſtances A B and C B, from the fulcrum B, may be as three to one, they will equiponderate; which is proved by applying the reaſoning at fig. 12 to the preſent figure.

B Since, of the three forces which act on a lever, the two which are applied at the extremes are always in a contrary direction to that which is applied in the ſpace between them; this laſt force will ſuſtain the effects of the other two: or in other words, if the fulcrum be placed between the weights, it will be acted upon by, or will ſuſtain their ſum; but if the weights are on the ſame ſide of the fulcrum, it will be acted upon by their difference.

C On the principle of the lever are made, ſcales for weighing different quantities of various kinds of ſubſtances; the ſteelyard, which anſwers the ſame purpoſe by a ſingle weight, removed to different diſtances from the fulcrum on a graduated arm, according as the body to be weighed is more or leſs in quantity; and the bent lever balance, which, by the revolution of a fixed weight, increaſing in power as it aſcends in the arc of a circle,

indicates the weight of the counterpoiſe. A B C (fig. 15) is a bent lever, ſupported on its axis or fulcrum B in the pillar J H. At A is ſuſpended the ſcale E, and at C is affixed a weight: draw the horizontal line K G through the fulcrum, on which, from A and C, let fall the perpendiculars A K and C D; then if B K and B D are reciprocally in proportion to the weights at A and C, they will be in equilibrio, but if not, the weight C will move along the arc F G, till that ratio is obtained. It is eaſy to graduate the plate F G ſo as to expreſs the weight in E by the poſition of C.

The beam of the common balance is uſually a bent lever, with equal arms. Its property of coming to reſt in an horizontal poſition, when the extremes are equally loaded, is a conſequence of its being bent, or, which is the ſame thing, of its fulcrum being above the line, joining the two points on which the ſcales are ſuſpended. For it is evident, (fig. 16) that there is but one poſition in which the lengths of the arms A B, B C, referred to the horizontal line D E, can be equal, and that is when the points A and C are on the ſame level.

Balances that move with very little friction on the fulcrum, and are exactly equibrachial, are highly valued. But this laſt property is of leſs importance than is commonly imagined. For, if two balances be equally ſenſible, and one of them not equibrachial, it is certain, that if the ſtandard weight be placed in one of the ſcales of this laſt, and counterpoiſed, and the ſtandard weight be afterwards removed,

moved, any other body ſubſtituted in its place will have exactly the ſame maſs, if it be in equilibrio with the counterpoiſe. In fact, this is the beſt method of weighing, when great accuracy is required. Or if the weights be always put into one ſcale and the quantities into the other, theſe laſt will be proportionally true among each other, which is quite ſufficient in all philoſophical experiments.

G On this principle alſo depends the motions of animals, the overturning or lifting great weights by means of iron levers called crows, the action of nutcrackers, pincers, and many other inſtruments of the ſame nature.

CHAP. III.

OF THE AXIS AND WHEEL, AND OF THE PULLEY OR TACKLE.

H THE axis and wheel may be conſidered as a lever, one of the forces being applied at the circumference of the axis, and the other at the circumference of the wheel, the central line of the axis being as it were the fulcrum. Fig. 17 is a perſpective view of the inſtrument, and fig. 18 is a ſection of the ſame at right angles to the axis. Then, if A B, the ſemidiameter of the axis, be to B C, the ſemidiameter of the wheel, reciprocally as the power E is to the power F, the firſt of which is applied in the direction of the tangent of the axis, and the other in the direction of the tangent of the wheel, they will be in equilibrio (56, U).

(56, U). For A C may be conceived to be a lever, whoſe fulcrum is B, and whoſe forces applied at A and C are in the reciprocal proportion of their diſtances.

To this power may be referred the capſtan or I
crane, by which weights are raiſed, the winch and barrel, for drawing water out of wells, and numberleſs other machines on the ſame principle.

The pulley is likewiſe explained on the prin- K
ciple of the lever. The line A C (fig. 19) may be conceived to be a lever, whoſe arms A B and B C are equidiſtant from the fulcrum B. Conſequently the two equal powers E and F, applied in the directions of the tangents to the circle in which the extremities are moveable, will be in equilibrio (55, S). And the fulcrum B will ſuſtain both forces (58, B).

But in fig. 20 the fulcrum is at C, therefore a L
given force at E will ſuſtain in equilibrio a double force at F, for in that proportion reciprocally are their diſtances from the fulcrum (56, U; 58, Z).
Whence it appears, that conſidering E as a force, M
and F as a weight to be raiſed, no increaſe of power is gained when the pulley is fixed, as in fig. 19; but that a double increaſe of power is gained when the pulley moves with the weight (fig. 20).

A combination of pulleys is called a tackle, and N
a box containing one or more pulleys, is called a block.

A D B (fig. 21) is a tackle compoſed of four pul- O
leys; two of which are in the fixed block A, and the

the other two in the block B that moves with the
P weight F. Now, becauſe the rope is equally ſtretched throughout, each lower pulley will be acted upon by an equal part of the weight: and, becauſe in each pulley that moves with the weight a double increaſe of power is gained; the force by which F may be ſuſtained will be equal to half the weight divided by the number of lower pulleys. That is, as twice the number of lower pulleys is to 1, ſo is the weight ſuſpended to the ſuſpending force.

Q But if the extremity C (fig. 22) be affixed to the lower block, it will ſuſtain half as much as a pulley; conſequently the analogy will then be, as twice the number of lower pulleys, more 1 is to 1, ſo is the weight ſuſpended to the ſuſpending force.

It is for the moſt part more convenient to form tackles with blocks of the form exhibited in fig. 23.

R This reaſoning depends on the equal tenſion of the rope, and is therefore concluſive only when the tackle is wrought by a ſingle rope. In the ſyſtem of pulleys (fig. 24) the power increaſes in a geometrical ſeries, whoſe common ratio is 2, and number of terms equal to the number of pulleys. Thus, if a force be applied at A, it will be acted upon by half the weight F; if at B, by $\frac{1}{4}$; if at C, by $\frac{1}{8}$; and if at D, by $\frac{1}{16}$; &c. The reaſon of which is evident from what has been already ſaid.

S It is evident, that in the compoſition of forces, the force produced is leſs than the ſum of the compounding forces; A D (fig. 3) being always leſs than

than the sum of A C and A B. On the contrary, in the resolution of force, a gain of force is produced, which is exemplified in the following instance:

The rope E A C D F (fig. 25) is passed over the pulleys A and B, and under the pulley C. Equal weights are suspended at E and F, whose actions on C may be represented by the equal lines C J and C H. These forces compounded give C D, which will express the force exerted by the two weights E and F, tending to move C in the direction of the perpendicular, and is less than the sum of C J and C H. Consequently a weight G being applied at C, whose quantity is less than the whole quantity of E and F in the same proportion, will sustain their effects, and remain in equilibrio. Therefore, if we consider E and F, as producing by composition a force equal to G, a loss of force ensues; and on the other hand, if G be considered as producing forces by resolution equal to E and F, an increase of force is acquired.

The quantity of this increase or diminution is readily determined thus.

From J let fall the perpendicular J K upon C D, then C K will be the half of C D. And J C is half the sum of J C and C H. Now, as the whole of that sum is to C D, so is the sum of the weights E and F to the weight G (for they respectively represent the forces of those weights) and so is J C to C K. But J C is the secant of the angle formed between the rope A C and the perpendicular C D, the line C K being radius. Therefore, as the se-

cant of the angle formed between one of the ropes and the perpendicular is to radius, ſo is the ſum of the weights E and F to the weight G.

v Hence it follows, that the general deduction, concerning pulleys and weights are only true when the ropes are parallel.

The pulley or tackle is of ſuch general utility, that it is needleſs to point out any particular inſtance.

CHAP. IV.

OF THE INCLINED PLANE, AND OF THE WEDGE.

w THE inclined plane has in its effects a near analogy to the lever. Let A B be an horizontal plane on which the weight E is placed, and let E D repreſent the force exerted by the weight. A B may alſo be conceived to act as the arm of a lever, whoſe fulcrum is A. Let this lever revolve on its fulcrum from B to C, then the weight E will be found at e, and will act on the plane A C with an oblique force e d, equal and parallel to E D. Reſolve e d into e b perpendicular, and b d parallel to A C, and the force e b will be deſtroyed by the reaction of the plane. With the other force b d, the weight will proceed with
x an accelerated motion towards A. Whence it may be obſerved, that the inclined plane, acting againſt e in the manner of a lever, deſtroys that force which is exerted in the direction of the tangent of its line of motion, and that the acting force in this inſtrument is that which in treating of the lever was

was rejected (56, v), as having no effect. The Y
force with which any weight on an inclined plane tends downwards in the direction of the plane, is to the weight itſelf, as b d to d e. Or as e F to A e, which is the ratio of the height of the plane to its length, becauſe the triangles b e d and F e A are ſimilar. But e F is to A e as the ſine of the angle the inclined plane makes with the horizon is to radius. See fig. 26. Therefore, as the ſaid ſine Z
is to radius, ſo is the force tending downwards in the direction of the plane to the weight. And becauſe radius is a conſtant quantity, the forces by which the ſame weight tends downwards in the directions of various planes will be as the ſines of their inclinations.

This inſtrument is not much uſed in its ſimple form.

If it be required to ſhew what force in the di- A
rection e p parallel to A B (fig. 27) will ſuſtain the weight e in equilibrio. Set off e m equal to b d, which will repreſent its force or tendency in the direction of the plane, and equal, but on the contrary ſide, ſet off e n, which will repreſent the force that, applied in the oppoſite direction, will ſuſtain the weight in equilibrio. Draw n p perpendicular to A c and e p parallel to A B, interſecting n p in p, e p will be the force required; for it is compoſed of e n and n p, and n p being perpendicular to the direction of the tendency of e avails nothing. Join p d and this laſt found force is to the whole weight of e, B
as p e to e d, or as e F to F A, which is the ratio of the

the perpendicular height of the plane to its horizontal baſe, for the triangles p e d and e F A are ſi-
C milar*. And ſince action and reaction are equal, and in contrary directions (22, R), it is evident that the ſame force e p, which ſuſtains e on the fixed inclined plane C A B, applied in the contrary direction would, if the plane be ſuppoſed moveable in the direction of its baſe A B, and the body e fixed by the application of an obſtacle q r, ſuſtain the effort with which the ſaid bo^dy tends to impel the plane from e towards p.

D The wedge is compoſed of two inclined planes joined together at their common baſe, in the direction of which the power is impreſſed.

E Let A B C (fig. 28) repreſent a wedge, whoſe vertex A is inſerted between the two bodies D and E, which being fixed in poſition, reſiſt in a certain degree any force which tends to ſeparate them. This reſiſtance uſually is, like the weight in the inclined plane, perpendicular to the baſe A F, and the power, or force employed to overcome it, is

* The ſimilarity of theſe triangles not being obviouſly deducible is proved thus:

Prolong p e and d m till they meet in s; and the right angled triangle m s e will be equal and ſimilar to the triangle n p e, s e being equal to e p.

The triangles s e d, p e d have the two ſides s e, e p equal, the ſide e d common to both, and the included angles s e d, p e d are both right angles. Conſequently the triangles are equal and alike in all reſpects. Euclid I. 4.

But it is eaſily ſhewn, that the triangle s e d is ſimilar to the triangle e F A, and ſo likewiſe muſt p e d.

impreſſed as was juſt mentioned, in the direction of the ſaid baſe. Therefore, by the property of the inclined plane, the force required to keep one half C F A of the wedge in equilibrio with the preſſure of the body D, is to that preſſure as C F to F A. But as the preſſure on the other half of the wedge acts with equal effect, a double force will be required to preſerve the equilibrium, that is, a force as C B to F A. Or, in general terms, in any F
wedge, as the line C B, joining the two equal ſides A B and A C, is to the diſtance between the vertex A, and the middle point F of C B, ſo is the force impreſſed to the reſiſtance in D and E.

This inſtrument is commonly uſed in cleaving G
wood, and was formerly applied in engines for ſtamping watch-plates. The force impreſſed is commonly a blow, which is found to be much more effectual than a weight or preſſure. This difference is uſually accounted for, by ſuppoſing that the tremulous motion produced by the ſtroke, conſiderably diminiſhes the very great friction at the ſides. But there is no doubt, that it is chiefly referable to the principles that obtain when reſiſting bodies are penetrated (37, M).

All cutting inſtruments may be referred to the H
wedge. A chizel, or an axe, is a ſimple wedge. A ſaw is a number of chizels fixed in a line. A knife may be conſidered as a wedge when employed in ſplitting, but if attention be paid to the edge, it is found to be a fine ſaw, as is evident from the much greater effect all knives produce by

a drawing ſtroke, than what would have followed from a direct action of the edge.

CHAP. V.

OF THE SCREW, AND OF MECHANICAL ENGINES IN GENERAL.

I THE Screw is compoſed of two parts, one of which is called the ſcrew, and conſiſts of a ſpiral protuberance, called the thread, which is wound or wrapt round a cylinder; and the other, called the nut, is perforated to the dimenſions of the cylinder, and in the internal cavity is cut a ſpiral groove adapted to receive the thread.

K Let A D G E (fig. 29) repreſent a cylinder, and A B C any flexible ſubſtance of a thickneſs altogether inconſiderable or evaneſcent. Suppoſe A B C to be a triangle, having a right angle at A, and one of the legs A B containing the right angle to be applied to the cylinder in a line parallel to its axis. Imagine now the cylinder to turn on its axis ſo that the triangle A B C may be rolled or wrapped cloſe on its ſurface. The lines B C, and all others, as I K, L R parallel to it, will then be contiguous to, or coincident with, the peripheries of circles, whoſe planes are all at right angles to the axis, and conſequently parallel to each other. But the line A C will become a curve A Q L M a N O P, &c.
L which is called an helix. This curve will always, or in every part, proceed from one towards the other

other end of the cylinder it enwraps, and will make equal angles with the generating circle of the cylinder. For any one of theſe angles, a k I, will be produced by the application of another angle A K I, always equal to the angle A C B.

Suppoſe the cylinder A D O E to be perpendicu- M
lar to the horizon, the lines B C and its parallels, together with all their correſpondent circles on the cylinder, will then become horizontal. Let the line A C now repreſent an inclined plane whoſe height is A B, and the helix being of the ſame length and height, and equally inclined to the horizon throughout, will not differ in mechanic effect from the inclined plane. That is to ſay, the N
tendency of a weight to deſcend on the inclined plane, will be exactly the ſame as on the helix.

Let A L be the perpendicular diſtance between O
two adjacent threads. Draw the horizontal line L R interſecting A C in R. Then L R will be equal to the circumference of the cylinder, and A R will be equal to one revolution of the helix. But A R repreſents an inclined plane, equivalent in power to the helix. Every helix therefore is equivalent in P
power to an inclined plane, whoſe length is equal to one revolution of the helix, and height equal to the diſtance between two adjacent threads meaſured by a line parallel to the axis of the cylinder contained within the helix.

If the horizontal thickneſs of the nut be diſre- Q
garded, it will not differ from a weight to be ſuſtained on the helical plane. Conſequently it R

will be kept in equilibrio by an horizontal force, which is to that of the weight, as the perpendicular diſtance between two adjacent threads is to the circumference of the cylinder (69, P. 65, A B).

S Or if the power be applied in the direction of the threads of the ſcrew, the equilibrio will be had when the power is to the weight as the perpendicular diſtance between the two adjacent threads is to the length of one thread of the ſcrew (69, P. 65, Y).

T But there are few, if any, inſtances where the ſcrew is uſed without the lever. If an arm E F (fig. 30) be applied to move the nut, the weight and the power may be conſidered as acting upon a lever, whoſe fulcrum is at the axis of the cylinder.

U And, therefore, the proportions laſt found (R, S) muſt be compounded with the ratio of the ſemidiameter of the cylinder to the diſtance of E from the axis of motion.

V It would be difficult to enumerate the very many uſes the ſcrew is applied to. It is extremely ſerviceable in compreſſing bodies together, as paper, &c. It is the principal organ in all ſtamping inſtruments for ſtriking coins, or making impreſſions on paper or cards, and is of vaſt utility to the philoſopher, by affording an eaſy method of meaſuring

W or ſubdividing ſmall ſpaces. A very ordinary ſcrew will divide an inch into five thouſand parts; but the fine hardened ſteel ſcrews, that are applied to the limbs of aſtronomical inſtruments, will go much farther. This method will be readily underſtood from the contemplation of fig. 31.

On

On the rule E G H F is fixed the upright piece E F. X
Through this piece the ſtem of the ſcrew A B paſſes, and is held by a collar, ſo that it may be moved on its axis without advancing or retiring in the direction of its length. The circular plate C D is fixed at right angles to the axis of the ſcrew which paſſes through its center. The piece I K M L is adapted to ſlide lengthways on the rule. This piece has a ſquare aperture, acroſs which is ſtretched a fine wire O P at right angles to the graduated line on the rule that may be ſeen beginning at N. At I K, on the ſliding piece, is a raiſed part perforated helically to receive the ſcrew. Suppoſe the ſcrew to have fifty turns in the length of an inch, and the edge of the plate to be divided into 100 equal parts: ſuppoſe likewiſe, that the wire O P ſtands between two of the dividing lines, and that it is required to determine its diſtance from one of them. Turn the ſcrew, which of courſe will move the ſliding piece, and obſerve, with a magnifier, when the wire accurately covers the dividing line. Then the number of whole turns of the ſcrew employed in the operation will give as many fiftieth parts of an inch, and the odd diviſions of the plate C D will ſhew the number of hundredth parts of a turn, that is to ſay, hundredth parts of one-fiftieth of an inch; or five thouſandth parts of an inch.

The ſcrew has been applied with great ſucceſs in Y
the diviſion of aſtronomical inſtruments*.

* By Mr. Ramſden, who has written a treatiſe on the ſubject.

Z It is easy to conceive, that when forces applied to mechanical instruments are in equilibrio, if the least addition be made to one of them, it will preponderate and overcome the effort of the other. But the want of a perfect polish or smoothness in the parts of all instruments, and the rigidity of all ropes, which increases with the tension, are great impediments to motion, and in compounded engines are found to diminish about one-fourth of the effect of the power.

A The properties of all the mechanical powers depending, as has been shewn, on the laws of motion laid down in the beginning of this treatise, and the action, or tendency to produce motion, of each of the two forces, being applied in directions contrary to each other, the following general rule for finding the proportion of the forces in equilibrio on any machine will require no proof.

B If two opposite forces be applied to the extremes of any mechanical engine, in the direction of the lines, in which, by the construction of the engine, the said extremes would move, and the intensities of the forces be to each other reciprocally as the velocities the extremes when put in motion would acquire in the same indefinitely small time: then those forces will be in equilibrio.

Suppose the forces to be weights, and the same may be expressed thus.

C If two weights applied to the extremes of any mechanical engine be to each other in the reciprocal proportion of the velocities resolved into a perpendicular

pendicular direction, (rejecting the other part) which would be acquired by each when put in motion for the ſame indefinitely ſmall time, they will be in equilibrio.

Whence it may be obſerved, that in all contriv- D ances by which power is gained, a proportional loſs is ſuffered in time. If one man, by means of a tackle, can raiſe as much weight as ten men could by their unaſſiſted ſtrength, he will be ten times as long about it.

It is convenience alone, and not any actual in- E creaſe of force, which we obtain from mechanics. This may be illuſtrated by the following example:

Suppoſe a man at the top of a houſe draws up F ten weights, one at a time, by a ſingle rope, in ten minutes. Let him have a tackle of five lower pulleys, and he will draw up the whole ten at once with the ſame eaſe as he before raiſed up one; but in ten times the time, that is, in ten minutes. Thus we ſee the ſame work is performed in the ſame time, whether the tackle be uſed or not: but the convenience is, that if the whole ten weights be joined into one, they may be raiſed with the tackle, though it would be impoſſible to move them by the unaſſiſted ſtrength of one man.

Or, ſuppoſe, inſtead, of ten weights a man draws G ten buckets of water from the hold of a ſhip in ten minutes, and that the ſhip being leaky, admits an equal quantity in the ſame time. It is propoſed, that by means of a tackle, he ſhall raiſe a bucket ten times as capacious. With this aſſiſtance he performs

performs it, but in as long a time as he employed to draw the ten, and therefore is as far from gaining on the water in the latter caſe as in the former.

H Since then, no real gain of force is acquired from mechanical contrivances, there is the greateſt reaſon to conclude, that a perpetual motion is not to be obtained. For in all inſtruments the friction of their parts and other reſiſtances continually deſtroy a part of the moving force, and at laſt put an end to the motion.

CHAP. VI.

OF THE CENTER OF GRAVITY.

I LET A B (fig. 32) repreſent a long ſlender body of an inconſiderable thickneſs, which is attracted by another body in the direction of the ſmall parallel arrows, a b c d, &c. Then the motion of A B will be the ſum of the motions of all the parts ſituate between A and B. Interpoſe the pointed obſtacle C D, and A B may be conſidered as a lever; C being the fulcrum. Conſequently, if C be ſo placed that the parts between A and C may be in quantity and diſtance from the fulcrum equipollent to thoſe between C and B, the whole body will reſt in equilibrio on the point C. This point is called the center of gravity.

K The thickneſs of A B being inconſiderable, the point C may be eſteemed as the center of gravity, but is not ſo when the thickneſs is taken into the account.

account. The foregoing illuſtration, beſides the advantage of its ſimplicity, may ſerve to ſhew that when we ſpeak of the whole attractive force of a body being collected in its center; as for example, the center of the earth, it is not to be imagined that any real power, or, as it were magic force, is ſuppoſed to exiſt in that center. In the ſame manner the body A B ceaſes to move, not immediately becauſe its center of gravity is ſuſtained, as if the cauſe of motion exiſted in that center alone, but becauſe, by the property of the lever, the forces on the ſide C B are made to counteract and deſtroy thoſe on the ſide C A.

The center of gravity is defined to be a point L
about which all the parts of a body or bodies are in equilibrio.

Therefore, the center of gravity of two bodies, M
A and B (fig. 33) will be a point C, in the right line that joins their centers of gravity, which is diſtant from the center of each body in the reciprocal proportion of their maſſes (56, U); that is, A C : C B : : B : A. And the center of gravity of three bodies, ſuppoſe A, B and E, will be found at D in the line C E, which joins the center of gravity of E with the point C. C D being to D E reciprocally as the ſum of the maſſes of A and B is to the maſs of E. For it is eaſily proved, ſuppoſing the lines to be levers, that the bodies A and B will equilibrate on the point C, which, as the fulcrum, will ſuſtain both their forces (58, B); and alſo, that the body E will equilibrate with the force ſuſtained at C; D being the

the fulcrum. In this manner the center of gravity of any fyftem of bodies may be found.

N Though the point called the center of gravity is defined from the univerfal property, gravity, yet, it may be as well defined from the inertia of matter; for the point C (fig. 32) is the center of inertia, or the point at which an impulfe will move the body without producing rotation; or, if the body were in motion in a right line, without rotation, the point C is the only point at which the oppofition of a fixed obftacle will deftroy the whole motion at once; but our intended concifenefs forbids the elucidation of rotatory motions.

O If two bodies move uniformly in right lines, their common center of gravity will either be at reft, or will move uniformly in a right line; and the fame is likewife true of the center of gravity of three bodies, for the center C (fig. 33) of any two of them may be confidered as one body. Therefore, if C and E be in motion, the common center D will either be at reft, or will move uniformly in a right line. And the fame may thus be fhewn of any number of bodies *.

P The common center of gravity of two or more bodies does not change its ftate of motion or reft from the mutual actions of the bodies upon each other; and therefore, the common center of gravity of all bodies mutually acting upon each other, is either at reft, or moves uniformly in a right line,

* Principia. in Corol. 4. ad 3. Legem motus.

actions

actions and impediments from without being excluded. For,

If the bodies A and B (fig. 33) act upon each other, the motion produced in each will be equal, (22, R) and the ratio of C A to C B will conſequently remain the ſame, whether they approach to, or recede from each other. The ſtate of C will not therefore be changed by their mutual actions. If the third body E be added to the ſyſtem, the center D, for the ſame reaſon, will not be changed, as to its ſtate of motion or reſt, whether E acts upon C or not: and the ſame may be proved of any number of bodies. Q

Since then the ſtate of the center of gravity of any ſyſtem of bodies, as to reſt, or uniform direct motion, is not affected either by the motions or mutual actions of the bodies of which it is compoſed, external actions or impediments being excluded, it is plain that the ſame law holds good in the motion of a ſyſtem of bodies as is obſerved by a ſingle body. For the progreſſive motion of a ſingle body, or of a ſyſtem of bodies, muſt be eſtimated by the motion of the center of gravity. R

Hence it is that the center of gravity of the earth is not affected by the motions on its ſurface, or in its bowels. When a projectile, a cannon ball, for inſtance, is thrown upwards, the projecting force reacting on the earth, cauſes it to move in the contrary direction; but as the motions are equal, the center of gravity remains the ſame. S

The

T The motions and actions of bodies upon each other in a ſpace that is carried uniformly forward, are the ſame as if that ſpace were at reſt.

U For the motions and actions of bodies upon each other depend on their relative motion, the velocity of which is the ſum of their abſolute velocities, when they are moved in oppoſite directions, or their difference when they move in the ſame direction. And this ſum or difference is not altered by an equal velocity impreſſed on all the bodies in the ſame or a parallel direction, as in the preſent caſe: ſince, when two bodies move in contrary directions, in a ſpace carried uniformly forward, the velocity added to that body, with whoſe motion the * motion of the ſpace conſpires, is exactly equal to the velocity deſtroyed in the other body, whoſe motion is oppoſed by that of the ſpace; and when the bodies move in the ſame direction, an equal velocity being added to, or deſtroyed in both, the difference is
V likewiſe unaltered. This is likewiſe confirmed by daily experience; motions performed on board a ſhip under ſail are the ſame as if the ſhip were at anchor; except ſo far as they may be diſturbed by the irregular toſſing of the waves, which affects them ſucceſſively, as much in one direction as another. A fleet of ſhips carried by an uniform

* Space being in its own nature immoveable, the expreſſion is here improper; but it conveys a clear idea of the propoſition in conciſe terms, though we can form no idea of bodies included in a ſpace being acted upon by that ſpace. The ſpace here mentioned is merely ideal, may be called relative, and is defined to be a moveable dimenſion.

current,

current, either preserve the same relative positions, or approach to, or recede from, each other in the same manner as they would if no such current existed. And the motions of bodies at the surface of the earth are no otherwise affected by its revolution on its axis than as the revolution is not rectilinear, the effects of which, though considerable, are not enough so to fall under common observation.

This proposition is likewise true, if the motion w
of the space be uniformly accelerated, or, which is the same thing, if all the bodies be constantly acted upon by parallel forces which act equally, according to their masses, on each of them.

For such forces will cause all the bodies to move x
with the same acceleration, and to describe equal spaces in the same direction with each other. They will not therefore change their relative motions or situations.

CHAP. VII.

OF PENDULUMS.

Y THE bodies ſpoken of in the preſent chapter are ſuppoſed to move without rotation, friction, or reſiſtance from the air, or any other medium; neither are the magnitudes of the bodies brought into conſideration.

It has already been ſhewn (65, Y), that the force of a body to deſcend along an inclined plane is to the whole force of its gravity as the height of the
Z plane to its length. If the body be at liberty to deſcend on the plane, the firſt-mentioned force will, by its conſtant and equal action (27, D) produce an uniform acceleration.

The ſpaces deſcribed from the beginning on a given inclined plane are (29, O) as the ſquares of
A the times; that is to ſay, the times of deſcription of inclined planes of the ſame inclination are as the ſquare roots of their lengths.

B The final velocities (36, H) of accelerated motions being equal when the forces are inverſely as the ſpaces paſſed through, and the length of an inclined plane being to its height in this ſame ratio of the forces, by which a body would deſcend along the plane, or fall freely through its height, it fol-
C lows that the final velocity acquired by a body that deſcends along ſuch a plane is equal to the final velocity it would acquire by falling freely through
D its height. Hence alſo the final velocity is always equal

equal when a body has fallen through an inclined plane of a given height, whatever may be its length.

The times of acquiring this given velocity, or of E
paſſing over the whole lengths, will be inverſely as the forces (35, G); that is to ſay, directly as the lengths when the heights are equal (65, Y).

Bodies that deſcend from a given height al- F
ways acquire the ſame final velocity, whether they deſcend along a ſingle plane or many. Let E G (fig. 34) repreſent an horizontal, and A G a perpendicular line; and ſuppoſe a body to deſcend along the inclined planes A B, B C, C D, D E; continue A B to F in the line E G, and draw the lines B K, C I, D H, parallel to the horizon. The body after paſſing through A B and B C will have acquired a velocity equal to the velocity it would have acquired ſimply by deſcending along B C or (80, D) along B L, added to the velocity it had at B: therefore the velocity at C, after paſſing through the planes A B, B C, is the ſame as would have been acquired by deſcending from the ſame height A I through a ſingle plane A L. The ſame reaſoning may be extended to prove, that when the body has arrived at D, it will have the ſame velocity as it would have acquired by deſcending in one plane A M of the ſame height A H: and ſo forth for any number of planes whatever.

Since the planes along which a body may paſs, in deſcending from a given height, are not limited either in number or magnitude, we may aſſume them to be indefinitely ſmall, and indefinitely numerous.

They may then be conceived to form a curve, and
G it will follow, that the laſt acquired velocity of a
body that deſcends by a gravity from a given height
along a ſurface either plane, polygonical or curved,
is always the ſame, and is equal to that velocity
it would have acquired by deſcending from the
ſame height by the action of its gravity in free
ſpace.

H On the planes of the ſame inclination the times of
deſcent from the beginning are as the ſquare roots
of the lengths of the planes (80, Z. 29, G).

I If a body deſcend along any number of inclined
planes, A B, B C, C D, D E, (fig. 34) and another body
deſcend along a like number of planes, N O, O P, P Q,
Q R, (fig. 35) having reſpectively the ſame incli-
nation and proportional lengths, namely, N O to
A B as O P to B C, and as P Q to C D, &c. then the
times of deſcent will follow the ſame law as would
have obtained if each had paſſed down a ſingle
K plane of the ſame inclination: that is to ſay, they
will be as the ſquare roots of the lengths paſſed
over. For, if the bodies were each to deſcend ſin-
gly along any two correſpondent planes A B and
N O, the times would be as the ſquare roots of thoſe
lengths (82, H): and the final velocities would be
in the ſame ratio (29, G). The ſame will be true
of the planes B C and O P. But now, ſuppoſe each
body to have deſcended through two planes A B,
B C, and N O, O P, then the time of deſcent through
B C will be leſs than it would have been in pro-
portion as the velocity it has at B is greater, namely,
in

in proportion to the ſquare root of A B (29, G). And in like manner the time of paſſing ſingly through O P will be diminiſhed in proportion to the increaſe of velocity gained at O, or in proportion to the ſquare root of N O. But A B is to B C as N O to O P, and therefore the times of paſſing through B C and O P are diminiſhed in proportion to their magnitudes, and muſt continue to have the ſame ratio as before. Again, ſince N O is to A B as O P to B C, the ſum of N O and O P, namely, N P, will be to the ſum of A B and B C, namely, A C, in the ſame ratio; and the ſum of the ſquare roots of N O and O P, namely, the ſquare root of N P, will be to the ſum of the ſquare roots of A B, B C, namely, the ſquare root of A C, as the ſquare root of N O to the ſquare root of A B, or as that of O P to that of B C. And ſince it has been ſhewn, that the times of paſſing over the planes N O, O P, and A B, B C, are reſpectively proportional to the ſquare roots of their lengths, the whole times muſt be proportional to the ſquare roots of their ſums, or whole length, which was to be proved: and the ſame reaſoning will apply to any number of planes.

If the planes be indefinitely ſhort and numerous, L they may be conceived to form a curve, and a ſimilar aſſemblage of planes reſpectively, in proportion to the former, will form a ſimilar curve. The foregoing arguments will then prove, that bodies deſcending along ſimilar curve ſurfaces deſcribe them in times which are as the ſquare roots of the lengths of the curves.

M If a body be urged towards a given point, by a force which is proportional to the diſtance of the body from the point, it will always arrive at the point in the ſame time, whatever the diſtance may be. Let I (fig. 36) be the point; and ſuppoſe two bodies to be let fall from any two points, A and R at a finite diſtance from I. Join A R, A I, and R I; and imagine A I to be divided into an indefinitely great number of equal parts, which, conſequently, will each be indefinitely ſmall. Through each point of diviſion in A I, imagine the lines B Q, C P, D O, &c. to be drawn parallel to A R, and they will divide the diſtance R I into an equal number of indefinitely ſmall parts. Any two intervals, A B and R Q, will be in proportion to the whole lines A I, R I, of which they are like parts. Now, the force may be eſteemed to be invariable or conſtant, while the body paſſes over the interval A B, becauſe the diſtance is not definitely leſs, during that time: and the ſame may be ſaid of the interval R Q; that is to ſay, the forces being as the diſtances A I, R I, will be alſo as the intervals or ſpaces A B, R Q, paſſed through: and the times of paſſing thoſe intervals will be equal (35, E). By the ſame argument the ſucceeding intervals B C, Q P, would have been paſſed over in equal times, if the motions had commenced at B and Q. But in the preſent caſe, the velocities already acquired in B and Q are (35, B) proportional to the ſpaces A B and R Q, and conſequently are ſuch as would, without any other action, carry the bodies over the ſpaces B C and Q P, in equal times.

It

It is therefore evident, that the bodies are carried over the ſecond ſpaces B C and Q P, by means of actions or forces which would have carried them reſpectively through thoſe ſpaces in the ſame time, increaſed by acquired velocities that would likewiſe have carried them through the ſame in equal times. The acquired velocities in B and Q muſt therefore ſubduct equally from the times in which B C and Q P would have been otherwiſe deſcribed, and the remaining or actual times of deſcription will be equal. This reaſoning will extend to prove, that all the other correſpondent intervals, C D and P O, D E and O N, &c. are reſpectively paſſed over in equal times; or, in other words, the whole lines A I and R I will be paſſed over in equal times; which was to be ſhewn.

If a number of indefinitely ſhort inclined planes be joined together, and the ſine of the inclination of any plane to the horizon (65, Z) be as its diſtance from the loweſt plane, meaſured along the planes, the tendency of a body to deſcend on them will be as its diſtance thus taken: conſequently (84, M) a body will, by paſſing along them from any diſtance, arrive at the loweſt point in the ſame time. N
The line in which the deſcent is made may be conceived to be a curve, becauſe no part of any definite magnitude lies in the ſame right line. This curve is termed a cycloid. O

Every thing that has been proved of the accelerated deſcent of bodies along inclined planes will hold good, mutatis mutandis (31, P) when bodies are P

retarded in their aſcent along the ſame, or congruous planes.

Q A pendulous body oſcillates by the ſame laws as it would move on an inclined ſurface of the ſame figure as the curve it deſcribes. For the reaction of the ſtring or rod is exerted againſt, and deſtroys the very force that ſuch a plane would deſtroy.

R The pendulums of clocks uſually vibrate in the arcs of circles. It has formerly been thought an advantage to make them vibrate in the arcs of cycloids; but the difficulties that attend the practical application are ſuch, that there is good reaſon to think that they produce greater errors in the admeaſurement of time than thoſe they are intended to remedy *. For this reaſon, we ſhall here only explain the properties of bodies vibrating in circular arcs.

S Let B (fig. 37) be a body pendulous from A, and moveable in the arc B F E, whoſe loweſt point is F: draw the line A F C, on which from B let fall the perpendicular B D. This laſt line B D will be horizontal. From B draw B C at right angles to A B, and conſequently touching the arc B F E. The body at B will then be urged towards F by its gravity in the ſame manner as if it reſted on an inclined plane B C, making the angle D B C with the horizon. Which angle, by reaſon of the ſimilarity of the triangles D B C, D A B, is equal to the angle D A B, or the in-

* The excellent Huyghens has explained the vibrations of pendulous bodies in his Treatiſe de Horologio Oſcillatorio; and the ſame thing is performed more generally in the Principia I. § 10.

clination

clination of the pendulum-rod or ſtring with the perpendicular. But the force at B is as the ſine of this angle (65, Z). Now, in very ſmall angles, the ſines, and conſequently the forces, are nearly proportional to the ſubtending arcs to be paſſed over: therefore, (85, N) all the circular vibrations T of the ſame pendulum are nearly equal when the arcs of vibration are ſmall.

But, in fact, becauſe the arcs increaſe faſter than U the ſines, the leſs vibrations of the ſame pendulum are performed in leſs times.

The times of the vibrations of pendulums that V deſcribe ſimilar arcs of circles, or, which is in fact the ſame, have equal angles of vibration, are (83, L) as the ſquare roots of the lengths of the arcs; and, becauſe the ſimilar arcs are as the radii they are deſcribed with, the times will be as the ſquare roots W of the lengths of the pendulums. This propoſition X may be affirmed likewiſe of all angles of vibration (87, T) when they are ſmall.

Thus far, in our reaſoning, we have ſuppoſed the Y force of gravity to be invariable in a given body; but if it be ſuppoſed to vary, its effects will vary in the ſame ratio (35, D, E); and the lengths of ſimilar arcs deſcribed by a pendulous body in equal times will be directly as the forces of gravitation that urge them. Now, the lengths of the arcs are as their radii, that is to ſay, the lengths of pendulums, vibrating through ſmall arcs (87, X) and meaſuring equal times, are as the gravitating forces.

Z If the ball B (fig. 37) of a pendulum were let go from any point remote from the loweſt point F, it would deſcend to F with an accelerated motion through the arc B F, and if its motion were without all reſiſtance or impediment, it would by a retarded motion preciſely ſimilar to its former acceleration, aſcend through an arc F E, equal to B F. From the point E it would again deſcend by acceleration to F, and again riſe to B by paſſing through the arc F B in the ſame manner as it before aſcended through the arc F E: the oſcillation would thus continue for ever, the angles and the times of all the vibrations being equal. But this cannot be; for there is no avoiding a certain degree of friction at A, and the ball B ſtriking, and giving motion to the parts of the air oppoſed to its courſe, muſt itſelf loſe as much motion as it communicates, (22, R) by which means the motion muſt continually decay, and at laſt become inſenſible.

A The pendulums of clocks are maintained in their motion by the action of the wheels, which are driven round by means of a weight or ſpring. Let E G F (fig. 38) repreſent the ſwing-wheel of a clock that is urged to revolve in the direction E G F; let C and D repreſent the pallets moveable on an axis at A, and ſo connected with the pendulum A B, that they are made to vibrate along with it. Suppoſe the ball to vibrate from R to B, one of the teeth of the wheel reſting againſt the pallet C, whoſe figure may be ſeen enlarged at I K L. When the pendulum returns

returns towards Q, the pallet C is drawn out, the wheel preſſing firſt along the plane I K, and afterwards on the inclined plane K L; by its action on which laſt, it puſhes the pallet or aſſiſts its motion, till at length the tooth ſlips off the point L. During this time the pallet D, whoſe figure is ſeen at P N O, is carried in between the teeth on the other ſide of the wheel; that when the wheel eſcapes the pallet C, another tooth drops on the plane N P of the pallet D. The returning vibration again draws out the pallet D, the tooth of the wheel aſſiſting its motion by preſſing along the inclined plane N O till it eſcapes at O. At this inſtant the pallet C has acquired its original ſituation, and therefore receives the adjacent tooth: and the whole proceeds as before. This is a very ſimple and good eſcapement. It is called the dead-beat eſcapement, becauſe the ſecond hand in clocks of this conſtruction falls with a ſingle or vulgarly a dead ſtroke on the diviſion line of the dial, and does not recoil, but remains motionleſs during that part of the vibration in which the tooth of the crown-wheel reſts on the plane I K, or P N of the pallet.

There are many other eſcapements which our preſent purpoſe will not admit of deſcribing. The leading requiſite of a good eſcapement is that the impulſe communicated to the pendulum, ſhall be invariable notwithſtanding any irregularity or foulneſs in the train of wheels.

If the reſiſtances ariſing from the friction at the B moving

moving parts, and from the motion communicated to the air, were always the ſame, and the clock were urged by a weight, the action of the ſwing-wheel on the pallets would be always the ſame at a given place, in conſequence of which, the figure of all the parts being ſuppoſed invariable, the arc of vibration would be conſtantly of the ſame magnitude, namely, ſuch as, that the motion loſt by the reſiſtances oppoſed to the pendulum ſhould be accurately equal to the motion communicated by the pallets, and the times would be equal; that is to ſay, the clock would be perfect, and would meaſure time accurately. But theſe conditions are not eaſily obtained. It is not found, that the variation in the reſiſtance of the air, ariſing from its changes of denſity, occaſions any ſenſible irregularity in clocks. The moſt conſiderable irregularities in the movement ariſe from the tenacity of the oil applied to the moving parts. For the oil is leſs fluid in cold than in hot weather; and when it is leſs fluid, a greater quantity of the maintaining power muſt be loſt in overcoming its rigidity: whence it muſt happen, that the teeth of the crown-wheel will in that caſe act leſs forcibly on the pallets, and the vibration will be leſs. If the pendulum be ſuſpended on an axis, this cauſe, together with the conſtant wear, is very noxious; but this defect is remedied by ſuſpending it by a ſtrait flexible ſpring. And in addition to this, there have not yet been found any materials

 which

which do not expand by heat. The pendulum rod therefore is longer in hot than cold weather, and the clock consequently goes slower (87, w).

It has not been determined from experiment and obfervation, how far thefe caufes refpectively affect the regularity of the clock's going. There are many good inventions for obviating their effects. C

The irregular action of the maintaining power, in confequence of its giving motion to the whole train of wheels, is rendered of no confequence by means of the efcapements, which are called detached or free efcapements. For in thefe the impetus of the fwing-wheel is not fuffered to act on the pendulum, but is employed in raifing a fmall weight in each vibration, which, by its fall, always gives the fame impulfe to the pendulum. D

The expanfion or contraction of deal-wood lengthways, by change of temperature, is fo fmall, that it is found to make very good pendulum-rods. The wood called fapadillo is faid to be ftill better. There is reafon to believe, that the previous baking, varnifhing, gilding, or foaking, of thefe woods in any melted matter, only tends to impair the property that renders them valuable. They fhould be fimply rubbed on the outfide with wax and a cloth. In pendulums of this conftruction the error is greatly diminifhed, but not taken away: but there are a confiderable number of ingenious contrivances for entirely removing it. E

The combination of metallic bars in the gridiron pendulum F

pendulum ſeems to be the moſt ſimple and effectual contrivance for this purpoſe, and is therefore the only one we ſhall here deſcribe. From the point of ſuſpenſion A (fig. 39) proceeds a ſmall flexible ſpring, by the alternate flexure of which the vibration is allowed to be made. The lower part of the ſpring is fixed to the frame B C, out of which proceed five equal cylindrical bars of metal. The two outer bars are ſteel, as is likewiſe the middle bar: the other two are a compoſition of zink and ſilver. The two outer bars are faſtened in the piece B C by means of pins, but the three inner ones paſs looſely into holes in the ſame piece, without being faſtened at all. The three inner bars paſs through a piece D E, near their upper ends, in which they are all faſtened by pins. At the lower extremities of the bars is another croſs piece F G, in which all the bars are faſtened by pins, except the middle bar that paſſes freely through, and carries the ball or lens H.

The conſideration on which this conſtruction depends is, that a given increaſe of temperature will cauſe the bars of zink and ſilver to expand about twice as much in their linear dimenſions as the bars of ſteel. To ſimplify our explanation, let us attend only to the ball H, without regarding the weight of the other parts. It will then only be required, that the diſtance between A and the ball H ſhould continue unaltered in every change of temperature, which is accompliſhed thus. Imagine the

the outer bars of ſteel to expand by heat, and they will ſuffer the frame F G to deſcend; the middle ſteel-bar will likewiſe expand in the ſame ratio of its length; and the ball H would conſequently be removed farther from the point of ſuſpenſion A if no other parts of the apparatus were affected by the heat: but the ſame heat expands the bars of zink and ſilver much more in proportion to their lengths, and therefore the adjuſtment may be ſo made as that the expanſion of theſe ſhorter bars may be equal to that of the longer ones of ſteel; that is to ſay, the expanſion of theſe two bars may be ſuch as to remove the croſs piece D E farther from F G, ſo as to raiſe the ball H upwards through a ſpace exactly equal to the quantity of the expanſion of the central, and the two outer bars. Whence the ball H will always be kept at the ſame diſtance from A.

It may be obſerved, that the two outer ſteel-bars F anſwer the purpoſe of a ſingle bar with regard to the expanſion, as do likewiſe the two next adjacent to the middle bar. Theſe bars, namely, two of ſteel and one of zink and ſilver, would have been ſufficient in theory, but the neceſſity of binding them together in that caſe produces a degree of friction of the parts that is, not without reaſon, thought to have a bad effect, by cauſing the action to take place by ſtarts. The adjuſtment of the contrary expanſions is made by pinning the piece D E at the various diſtances from F G, by which means the ratio of the length of ſteel to that of the zink and

and ſilver may be altered in any convenient degree: and this adjuſtment being made from actual obſervation of the clock's going, it is evident, that no practical inconvenience can reſult from our gratuitous ſuppoſition of all the parts, except H, being without weight.

CHAP.

CHAP. VIII.

OF THE MOTION OF A BODY, WHICH IS ACTED UPON BY A CENTRIPETAL FORCE.

If a body at A (fig. 40) be carried with an uniform direct motion in a given line A E, and rays be drawn from the equidistant points, A, B, C, D, E, to any point L, without the line A K, the areas A L B, B L C, C L D, D L E, &c. will be equal to each other *. I

And these areas which are described in equal times, will not be altered by any centripetal force acting on the body A, and impelling it towards L. For,. K

Suppose the body A to describe the equal spaces A B, B C, C D, and consequently with respect to the point L, the equal areas A L B, B L C, C L D, in equal times. Let a centripetal force be impressed at D, which singly would cause it to describe the space D d in the same time as D E, which is equal to D C, &c. Complete the parallelogram D d e E, and (23, s) at the end of the time the body will be found at e; having described the diagonal D e. The tri- L

* This reasoning depends on that well-known proposition (Euclid I. 38) that triangles constituted upon equal bases, and between the same parallels, are equal to one another.

angle

angle D e L will then be equal to D E L, because both stand on the same base D L, and between the parallels D L and E e. Continue e F equal to D e, and the area e F L will be equal to D e L, for the same reason; e F representing the space which would be described in the same time as D e, if no new impulse were given at e. Let a force e m be impressed at e, and by the same process it is proved, that e f L is equal to e F L. The like may be proved of the triangles f g L, g h L, &c.

M Since therefore any single impulse can only alter the velocity and direction, but never affect the area described, it is plain that any number of successive impulses will likewise have no effect in altering the area. Suppose the number of impulses to be infinite, or, in other words, let a force directed to the center act continually on the body, and a polygon with an infinite number of sides, that is to say, a curve, will be described, whose radius accompanying the moving body, will describe or sweep over equal areas in equal times.

N And conversely, if a body revolve about a point, so as to describe a curve, whose radius accompanying the body, shall sweep over equal areas in equal times, the centripetal force which deflects the motion from a right line, must be directed to that point.

O But no instance of a centripetal force directed to an immoveable point is found in nature. Bodies attract one another, and that mutually. Therefore,

fore, if one body revolves about another, this laſt will not remain at reſt, but will revolve in a * ſimilar curve about the common center of gravity, as will alſo the firſt body. That is to ſay, if the center of gravity be at reſt, the two bodies will abſolutely move in ſimilar curves about that center, and relatively about each other in curves ſimilar to thoſe laſt mentioned.

These motions will not be altered, if the center of gravity be ſuppoſed in motion (78, T. 79, W). P

Therefore, when we ſpeak of the orbits and periodical revolutions of bodies, we may in general regard one of the bodies as ſtationary, and the other as revolving round it. Q

If a body revolve round a center in an orbit which is not circular, it is plain, that to deſcribe equal areas in equal times, it muſt move ſwifter when near the center than when more diſtant; and it is likewiſe evident, that when the velocity, and conſequently the tendency to fly off in a tangent is increaſed, a greater centripetal force will be required to retain it in its orbit. R

From the properties of the ellipſis it is demonſtrated, that a body revolving in that curve, whoſe centripetal force tends to one of its foci, muſt in any part of its orbit be attracted towards that focus, by a force which is reciprocally as the ſquare of its diſtance †. S

* Principia, I. 57. † Principia, I. 11.

 A very

T A very complete and clear idea of the ellipſis may be had from the common way of deſcribing it; if a thread C A C (fig. 41) be faſtened by its ends at the points C C, and a pointed inſtrument be inſerted in the bight or bend at A, and moved towards B or E, keeping the thread at full ſtretch, it will in one revolution deſcribe the ellipſis A B D E. C C are called the foci.

U To illuſtrate this doctrine of revolving bodies, we may obſerve, that as gravity conſtantly acts on all bodies in the vicinity of the earth, attracting them towards its center, every projectile, which is not thrown in the line of the perpendicular, may be conſidered as a body revolving about that center; and if its orbit be not ſufficiently large to contain or circumſcribe the body of the earth, it will be interrupted in its courſe, and remain at reſt ſomewhere on the ſurface. Thus let A B E (fig. 42) repreſent the earth, whoſe center is at C; then if a body be projected from A in the direction A F, it will by the action of the centripetal force, be deflected into the curve A G B, and will remain at reſt at B, being prevented from deſcribing the whole orbit A G B D A, by the body of the earth, which interrupts its courſe at B. But the part A G B of the elliptical orbit of a projectile is ſo ſmall, in compariſon to that part which is not deſcribed, that it may without any ſenſible error be conſidered as a parabola, except ſo far as the reſiſtance of the air, which is not here regarded, makes it fall ſhort of B, by deſtroying part of its motion.

The orbit A G B D A, of which the parabola is a part, would have been described upon the supposition, that the attraction towards the center continues to observe the same law within as without the sphere. But this supposition, however, is not true; for a sphere of uniform density, composed of particles which attract each other with forces reciprocally as the squares of their distances, will attract bodies without its surface according to the same law; relation being had to its center. But the centripetal forces of bodies placed within the sphere, will be directly as their distances from the center *.

Let the circle B C D G (fig. 43) represent the earth. From the top of the mountain A, let a body be projected in the horizontal direction A F, with a force that will carry it to B on the surface. Imagine it to be projected in the same direction with a still greater force, and it will be carried to C. A still greater increase of force will carry it to D. And a yet greater augmentation will carry it round the earth to A, where it will proceed with a velocity equal to that with which it was first projected, and by consequence, the resistance of the air being disregarded, will revolve in that orbit for ever. But if the projectile force be still more increased, it will describe the ellipsis A b A with an unequable motion; slower at b and swifter at A, and continue to revolve for ever in that orbit.

* Principia, I. 83.

If gravity acts in the diſtant ſpaces of the heavens inverſely according to the ſquares of the diſtances, it will be eaſy to apply this to the motions of the celeſtial bodies. We ſhall again reſume this ſubject; but in the mean time it is neceſſary, that the appearances ſhould firſt be deſcribed before an explanation of them can be given.

BOOK I.

SECT. III.

Aſtronomy.

CHAP. I.

CONCERNING THE SYSTEM OF THE UNIVERSE.

THERE are two methods by which knowledge 1
may be acquired and announced; namely, by analyſis, or by ſyntheſis. In the method of analyſis the proceſs is made from things that are compounded to things that are more ſimple. Cauſes, or firſt principles, are inveſtigated by attending to, and examining their effects. But in the method of ſyntheſis the proceſs is directly the contrary; for here the cauſes, or firſt principles, being known or aſſumed, are made by compoſition or combination to account for their effects. It is very manifeſt, that in the acquiſition of philoſophical knowledge the former method muſt be firſt made uſe of. We ſee no ſimple events in nature, and we cannot come at cauſes but by analyſing the effects we behold. Thus it is that firſt prin-

ciples are obtained, which may afterwards be extended by ſyntheſis, to account for other phenomena in a more univerſal manner. Generally ſpeaking, the analyſis appears beſt adapted for acquiring knowledge, while the ſynthetical method is more convenient and conciſe for communicating it, when known.

In the communication of the knowledge that relates to the heavenly bodies, and is termed aſtronomy, we might aſſume as eſtabliſhed firſt principles every thing which reſpects their mutual poſitions and motions abſolutely conſidered, and from thence deduce ſynthetically the phenomena that would appear to a ſpectator placed on the earth or elſewhere. Or we might ſtill more generally, from the laws of motion aſſumed as firſt principles, deduce the conſequences that would ariſe from the motions of bodies in circumſtances ſuch as the heavenly bodies are known to be placed in. By theſe methods the ſcience would be moſt expeditiouſly taught, and even ſuppoſing the firſt principles to be merely aſſumed at hazard, or gratuitouſly, yet their conſtant agreement with the events that happen in nature might furniſh no inconſiderable preſumption of their truth. However, it is clear, that the knowledge on which the ſynthetical reaſoning is built, muſt be either aſſumed gratuitouſly, or obtained by analyſis. It is not eaſy, nay, it is perhaps impoſſible, to form any conception how a finite intelligence can, to any advantage, make

make use of first principles taken without a previous use of the analytical method of deduction. Yet the whole history of natural philosophy affords numberless instances of men of real abilities who have indulged their vanity and indolence in deducing consequences from principles entirely hypothetical, and often false. Far from wishing to imitate these, we shall not assume, even established truths, without giving their proofs where they can be explained with that degree of facility which our intention demands. Instead, therefore, of deducing the apparent phenomena from the real motions of the heavenly bodies, it is presumed that it will be much more interesting, though rather more prolix, to note the obvious appearances, and thence infer their causes. These inferences have been the consequence of the observation and study of several of the most distinguished men of genius in the course of many ages. The truth has been acquired by slow degrees, and indirect methods. It has often been obscured by the admixture of error. Its progress has been retarded by the operation of prejudice, and the pride of false science. When discoveries are completed, it is easy to trace the most direct steps by which they might have been made, though it has scarcely ever happened that the discoverers proceeded by those steps. Let us therefore pass over in silence the various and intricate schemes made use of to solve the celestial appearances before the ancient system of the world

was revived by Copernicus*, and since established for ever by the immortal Newton. Let us imagine ourselves in the open air busied in the contemplation of the phenomena that occur in the heavens, and while we note the facts, and make plain deductions from them, we shall be insensibly led to the knowledge of the beautiful regularity and order which prevail through the immense regions of space, and evince the intelligence and power of the First Cause.

The first and most obvious phenomenon that presents itself to observation, is the apparent diurnal motion of the visible sphere of the heavens; by which the sun, moon, and stars are seen to rise and set. This motion is observed to be subject to seeming irregularities. If its period be estimated from sun-rise to sun-rise, a little time shews, that the sun does not always rise at the same point, nor remain above the horizon so long in winter as in summer. The moon is still less adapted to the purpose of determining this period, its variations being in every respect more conspicuous. The stars remain, which appear indeed to rise, and set regularly, but yet in a period shorter than the natural day; for those

* A. D. 1543, the year of his death. After suppressing his book, "de Revolutionibus orbium celestium," for more than thirty-six years, it was at length published, and a copy brought him a few hours before his death. Gassendus in vita Copernici. See also Sir John Pringle's elegant "Discourse on the Attraction of Mountains."

stars,

ftars, which at a certain time of the year are feen to rife at midnight, are found to make their appearance early in the evening, after the fpace of three months is elapfed. It is therefore to be determined which of thofe motions ought to be regarded as the motion of the heavens; and it is much more obvious and intelligible, to fuppofe, that the fun, by a relative motion to the eaftward with refpect to the fixed ftars, fhould make the days fomewhat longer than the real time of a revolution, than that all the ftars, while they preferve their mutual diftances unaltered, fhould conftantly move with a velocity greater than that of the fuppofed celeftial fphere. To determine this relative path of the fun is not difficult. By the fhadow of a perpendicular ftaff or other equivalent inftrument at mid-day, its varying declination towards the north or fouth may be known, and the advance in the rifing of the ftars will mark its difference in right afcenfion. By this, or fome fuch method, it may be difcovered, that while the fixed ftars rife and fet, each on its proper points of bearing or pofition, without varying their relative fituations; the fun, by defcribing annually a circle towards the eaft, inclined to the direction of its daily courfe in an angle of $23\frac{1}{2}$ * degrees, muft occafion all the difference of feafons, length of days, &c.

* Every circle is fuppofed to be divided into 360 parts, which are called degrees.

In

B In noting theſe appearances, it is natural to ſelect the brighteſt ſtars as objects of our attention. The planetary bodies will on this account be the early objects of our notice. The planet Venus eſpecially, receding from the ſun to the eaſtward, will appear as an evening ſtar in the weſt after ſunſet; and afterwards will diſappear on its re-approach to it, and be ſeen at a nearly equal diſtance to the weſtward, and riſing before the ſun, become a morning ſtar. The ſlowneſs of its apparent motion near its greateſt elongation or angular diſtance from the ſun, ſhews that it is moved in an orbit, near the center of which the ſun is placed; and the ſhort time employed in paſſing from its greateſt elongation eaſtward to its greateſt elongation weſtward, when compared with the time of its courſe between the ſame elongations in the contrary direction, ſhews that its revolution is made from weſt to eaſt. The proportion between its diſtance from the ſun, and that of the ſun from the earth, may be found from the quantity of its greateſt elongation.

C To illuſtrate this, let S (fig. 44) repreſent the ſun, E the earth, I M K N the orbit of Venus, C S D part of the ecliptic, or ſun's apparent annual path in the heavens. Then to a ſpectator at E, ſituated nearly in the plane of the planet's orbit, the planet when at B will be referred to the point b in the ecliptic; and when by its abſolute motion in its orbit, it has deſcribed the arc B U, it will appear to have deſcribed the arc b D. When at U, it will appear ſtationary at D, and after a little time begin

to

to move back from D to b: for after describing the arc U J, it will again be seen at b. Continuing its course, it will arrive at V, having apparently passed through the arc b c. At V it will again become stationary and afterwards move to G, F, A, &c. which will be represented by their corresponding points in the ecliptic. Now, since the motion, as seen from the earth, is that which appears in the ecliptic, and since the apparent motion from b to D may as well be produced by a real motion from J to U, as by one from B to U, it remains to be determined in what direction the real motion is made by which the apparent motion is produced. Now, because E C and E D are tangents to the orbit, the points D and C, which correspond with the positions U and V, are those of its greatest elongations; and because the arc U V, which is passed over in the inferior part of the orbit between the two greatest elongations, is less than the superior arc V A B U, which is passed over between the same elongations, it is plain, that when the planet is in the inferior part of the orbit, the space C D will be performed in less time than when it is in the superior part. It is also evident, when the planet moves in the superior part of its orbit, that the apparent motion in the ecliptic has the same direction as the real motion. Therefore, since we have a criterion to distinguish the motion in the superior from that in the inferior part, we can easily determine the direction of the motion in its orbit, which is proved to be from west to east.

The

E The diſtance of Venus from the Sun, in proportion of that of the Sun from the earth is determined from its greateſt elongation: thus draw the line V S, which will be at right angles to the tangent V E; then in the right angled triangle V E S, by the rules of plain trigonometry,

As radius

Is to the ſine of the angle of greateſt elongation V E S,

So is the Sun's diſtance from the earth E S,

To the diſtance of Venus from the Sun V S.

F By ſimilar obſervations on the planet Mercury, it is determined that its revolutions are performed round the ſun in the ſame manner, becauſe they are accompanied with circumſtances of the ſame nature as appear in the motion of Venus.

CHAP.

CHAP. II.

OF THE FIGURE AND MOTION OF THE EARTH.

We have not yet considered the effects which the sun's annual motion in the ecliptic has upon the apparent motions of the planets, though it is very considerable: neither have we determined whether this apparent annual motion be the consequence of a real motion of the sun about the earth, or of the earth round the sun. The celestial phenomena may be explained either way, but in a much more simple and intelligible manner by the latter supposition. We shall therefore previously give an account of the figure of the earth, and the reasons on which the supposition of its motion is founded. The proof will come more properly when we treat of the physical causes of these motions. At present we only describe appearances, and draw plain inferences from them.

The purposes of astronomy require, that the fixed G stars should be classed into constellations. When their relative situations are known, it must soon be perceived, that their diurnal revolutions are performed round an axis, obliquely situated with respect to the horizon, or circle that bounds our view; one of its extremities or poles being above the horizon to the * north, and the other as far below it to the

* In this elucidation the observer is supposed to be in north latitude.

south,

south, and consequently invisible. By travelling to the northward, the north pole is observed to become more elevated, and that exactly in proportion to the space travelled over; from which circumstance it follows, that the earth is round or spherical.

For, let I B E, fig. 46, represent a plain section of the earth at right angles to its surface; let A B represent the plumb line or perpendicular, and B P the line of direction, in which the * pole star is seen; the angle A B P will then be the complement of the altitude of the star: let the same star be seen from another point E in the line of direction E Q, which, by reason of the great distance of the star, may be esteemed parallel to B P; the angle D F Q will then be equal to the angle ABP, or complement of the former altitude at B. From E draw the plumb line or perpendicular E F; the angle D E F will then be the difference between the two co-altitudes ABP, F E Q, or between the two altitudes; but this difference is equal to the angle B C E, formed between the plumb lines, and is proportional to the arc or distance B E †. Now,

* There is no star situated at the pole. The star α in Ursa Minor, which is called the pole star, is about 2½ degrees distant from it.

† Let the part H N of the line O P (fig. 46) be comprehended between the perpendiculars A H, F N, which, if continued, meet at G. Conceive H N to be divided into an indefinite number of equal parts, by means of perpendiculars B I, C K, &c. severally prolonged till they meet. Then in the triangle H O I, the sides, H G, I O, are equal, because opposed to equal angles at H and I: and for a like reason, in the triangle

Now, if the angle formed between two perpendiculars to a given line be always in proportion to that part of the line comprehended between the perpendiculars, the line itſelf is circular: and that ſolid, whoſe ſections, paſſing through two oppoſite points, are all circles, is itſelf a ſphere.

Hence, if the length of the arc B E be meaſured, I
and its quantity in degrees known by obſervation on a ſtar, the length of the whole circumference of the earth may be found by this proportion. As the quantity of degrees is to the length meaſured, ſo is the whole circumference, or 360 degrees to its length.

The modern circumnavigation likewiſe proves K
the ſphericity of the earth; for, by ſailing continually eaſtward, or continually weſtward, veſſels arrive again at the port from whence their firſt departure was taken.

Alſo, in an eclipſe of the moon, the ſhadow L
of the earth is always projected in a circular form. Now, it is evident, that the body, whoſe ſhadow is in all poſitions a circle, muſt itſelf be a globe.

Unfurniſhed with thoſe proofs, which the ſaga- M
city and more accurate obſervations of later

angle I G K, the ſides I G, G K are equal. The ſame proceſs of argumentation will extend to prove, that all the ſmall triangles between H and N are iſoſceles, having a common vertex at G; that is to ſay, there is no definite part of H N from which a right line can be drawn to the point G, either greater or leſs than H G: H N is therefore a curve, having ſuch a relation to a certain point, that all right lines drawn from it to that point are equal; or it is circular. Which was to be ſhewn.

ages have afforded, the ancients could not adduce thoſe reaſons for the earth's motion, that depend on the general laws of motion, and the nature of gravity. Without doubt they had recourſe to thoſe which depend on the moral fitneſs of things. They were perſuaded that the wiſdom of the Creator had formed every thing in the beſt manner poſſible, and therefore, that when an effect could be as well produced by ſimple as by complicated cauſes, the obſerver of nature ought to attribute it to the former. They ſaw the two planets, Mercury and Venus, revolving round the ſun in orbits, whoſe radii are leſs than the diſtance between the ſun and the earth: the ſuperior planets, Mars, Jupiter, and Saturn, were alſo obſerved to move in orbits about the ſun, but at greater diſtances than that between the ſun and earth. If the ſun were ſuppoſed to move abſolutely in the ecliptic or its apparent path, it muſt carry the orbits of theſe bodies along with it, and conſequently their abſolute motions muſt be very complicated; but if the earth be ſuppoſed to deſcribe an orbit round the ſun, between Venus and Mars, the abſolute motions become ſimple and natural, and an admirable uniformity prevails throughout the ſyſtem.

The annual motion of the earth being allowed on this principle, its diurnal motion would follow by the ſame argument; it being much more reaſonable and conſiſtent to ſuppoſe, that the earth, by a daily revolution on its own axis from weſt to eaſt, ſhould occaſion the apparent motion of the celeſtial bodies, than

than that thoſe bodies ſhould, beſides their other various motions, have that aſtoniſhing velocity which a real diurnal motion would produce. The objections common obſervers might make would be eaſily diſproved by men whoſe penetration was capable of going thus far. From the obſervations by which the ſpherical form of the earth was diſcovered, they would alſo gather, that bodies fell not abſolutely down, or in a direction referable to pure ſpace, as was imagined, but always in a line directed towards the center of the earth, and conſequently that no danger of bodies falling off would ariſe from its continual change of poſition. The inſtances of ſhips carried by the tides in calm weather would likewiſe ſerve to ſhew, that the relative motions or poſitions of bodies are not changed by an equal velocity given to them in the ſame parallel direction.

CHAP. III.

OF THE MUTUAL APPEARANCES OF THE SUPERIOR AND INFERIOR PLANETS.

THAT the planets Mars, Jupiter, and Saturn revolve in orbits, which include the orbit of the earth, is evident, becauſe they are frequently ſeen in the part of the ecliptic directly oppoſite to the ſun; and that the orbits reſpect the ſun as a center, appears as well from thoſe oppoſitions which happen in every part of the ecliptic, as from their unequable

apparent motions, which are explained by referring them to that center.

P We have conſidered the apparent motions of the inferior planets as far as relates to their ſituation with reſpect to the ſun. The motion of the earth affects thoſe appearances, to ſpeak in general, only by prolonging the time they employ to return again to the ſame ſituation.

Q The earth at E (fig. 44) is a ſuperior planet with reſpect to Venus. A ſpectator on Venus at B would ſee the earth E elongated from the ſun under the angle E B S; which angle of elongation would increaſe by the motion of Venus in its orbit from B to U, where it becomes a right angle E U S. From J it would be ſeen in an angle of ſtill greater elongation E J S, and from M it would be ſeen directly in oppoſition to the ſun. Paſſing from M to K, V, &c. the angle of elongation would decreaſe till the arrival of Venus at N, whence the earth would be in conjunction with the ſun, and the angle of elongation would vaniſh. This relative motion of the ſuperior planet with reſpect to the ſun is contrary to the order of the ſigns, or from eaſt to weſt, and depends entirely upon the motion of the inferior planet on which the ſpectator is ſuppoſed to be placed.

R If the earth E was at a diſtance indefinitely great, the lines B E, U E, J E, &c. might be eſteemed parallel, and conſequently the ſpectator would behold it always in the ſame point of the ecliptic, its ſituation with regard to the ſun being varied

only

only by the apparent motion of the ſun, occaſioned by the real motion of Venus. But as this is by no means the caſe, an apparent motion of the earth among the ſigns of the ecliptic will be produced. Thus, the earth viewed from N, will appear among the fixed ſtars at P; from B it will appear at R; from U at O, where it will be ſtationary ſo long as the orbit of Venus does not ſenſibly differ from its tangent; from J it will be ſeen returned back to R with a retrograde motion; from M at P; from K at T; from V at Q, where it again becomes ſtationary; and from A it will be again ſeen at T, its motion having again become direct: whence we may obſerve, that

When a ſuperior planet viewed from an inferior S
appears ſtationary, the inferior planet viewed at the ſame time from the ſuperior is alſo ſtationary; and,

When the inferior planet viewed from the ſupe- T
rior moves apparently retrograde, or contrary to the order of the ſigns, the ſuperior planet has alſo an apparently retrograde motion.

But ſince the earth has an annual motion round U
the ſun in its orbit, (110, M) we are therefore to diſcover what part of the apparent motion of Venus is produced by that cauſe. It is plain, that if the earth were at reſt, and Venus ſeen at U, its greateſt elongation, it would again be ſeen in the ſame poſition, after performing a complete revolution in its orbit. But while Venus is performing this revolution, the earth is carried from E towards W, and ſo

forth. Therefore Venus must pass between two similar elongations, not only a complete revolution, but likewise the whole angular space which the earth has performed in the same time. Hence its periodical time may be found. For the time between two similar positions is observed, to be 583 days. Now, dividing the earth's orbit into 365 equal parts or days, the angular velocity of Venus will be denoted by the angular space passed over in the given time, namely, one revolution, or 365 days added to 583 days, equal to 948; and the earth's angular velocity will be 583.

v The periodical times of Venus and the earth will be reciprocally as their angular velocities; consequently,

As the angular velocity of Venus -	948
Is to the angular velocity of the earth	583
So is the periodical time of the earth	365
To the periodical time of Venus	224$\frac{1}{2}$

w Were it not for the fixed stars, it would be impossible to discover or observe the annual motion of the earth. We should conclude, that each planet made a complete revolution between any two similar situations with respect to the sun, because the spaces of elongation are similarly described, and are in quantity the same, whether the earth be in motion or not. Thus, if the earth be fixed at E, the same apparent elongations will be made by Venus with any velocity whatsoever in its orbit, but they will occur more frequently the greater the velocity. If a motion be given to the

 earth

earth in the orbit E W, Venus will approach from U to M, which is now in motion, with a velocity equal to the difference between its angular velocity and that of the earth: or if the earth's angular velocity be greateſt, it will apparently recede from M, and deſcribe its revolutions in the contrary direction to its real motion. Now, as all the apparent motion of Venus in elongation is known by its approach or receſs from the line S E, and ſince any angular motion of S E can only change the relative velocity of Venus; and ſince a change of velocity will not alter the elongations, except as to time, it is evident, that we cannot determine whether E be at reſt or no, from the appearances of the planets which revolve about the ſun. It is then from the apparent motion of the ſun, with reſpect to the fixed ſtars, that we conclude that the earth deſcribes an orbit in about 365 days.

If the ſuperior planet E be at reſt, the retrograde X
motion of the inferior planet U among the fixed ſtars will be the ſame as its motion in elongation, viz. the angle U E V. But if E move in the ſame direction as U, but angularly ſlower, the arc deſcribed by the retrograde motion in the ecliptic will be leſs than that deſcribed between the two oppoſite elongations. The ſame is true of the retrograde motion of the ſuperior viewed from the inferior planet.

For the motion of E towards W cauſes an ap- Y
parent motion of the ſun towards D. And as the retrograde motion of U referred to the arc DS is

ſloweſt near the elongations, it is plain that υ will not become ſtationary in the ecliptic till its apparent motion in elongation from D towards S is equal to the ſun's apparent motion in the contrary direction; that is to ſay, till ſome time after paſſing the greateſt elongation, ſuppoſe at d. After which its motion muſt become retrograde till it arrives at H, equidiſtant from its greateſt elongation on the other ſide, where it will again become ſtationary, its apparent motion in elongation being equal and contrary to that of the ſun in the ecliptic. Now, the angle H E d is leſs than the angle of retrograde motion in elongation C E D. And ſince the angle I E q is equal to H E D, it is alſo leſs than C E D. But thoſe angles I E q and H E D are the meaſures of the retrograde motions of the ſuperior and inferior planets, when viewed from each other. Whence the propoſition is evident.

CHAP. IV.

OF THE SUPERIOR PLANETS, AND OF THE TRUE FORM OF THE PLANETARY ORBITS.

z THE appearance of the earth when viewed from Venus being explained, it will be eaſy to apply that explanation to the apparent motions of the ſuperior planets. Of the two inferior planets Venus ſerved us as an inſtance; and of the three ſuperior ones we ſhall ſelect Jupiter, as being the moſt bright and conſpicuous. The motions of this planet being accounted for, ſimilar obſervations and ſimilar reaſoning will obviouſly ſolve thoſe of the other planets,

nets, whose particular phenomena will not, therefore, require a more minute elucidation.

That the planet Jupiter revolves in an orbit, A which includes that of the earth, and respects the sun as its center, was shewn in the beginning of the last chapter; and its apparent motions are observed to be similar to those which it was proved the earth would have when seen from Venus. It remains to discover its periodical time and distance from the sun.

Let S (fig. 47) represent the sun, E the earth, B J Jupiter, the circle E e a the earth's orbit, and the circle J j A the orbit of Jupiter. Suppose Jupiter to be in opposition to the sun. The earth revolving in its orbit will, in the space of 365 days, arrive again at E, but the opposition will not then happen, because Jupiter in the mean time will have moved in its orbit towards j. The earth must therefore pass through the arc E e or 33½ days before it overtakes it. Consequently,* the angular velocity of Jupiter will be denoted by 33½, and that of the earth by one whole revolution, (or 365) added to 33½, equal to 398½. But as the periodical times are reciprocally as the angular velocities, it will be

* As the angular velocity of Jupiter 33½
Is to the angular velocity of the earth 398½
So is the periodical time of the earth 365 days
To the periodical time of Jupiter 4340 days.

* Smaller fractions being rejected, the periodical times are not here exact.

C The periodical time of Jupiter being thus obtained, it will be easy to determine its * heliocentric place at any time before or after the opposition, and the proportion of its distance from the sun to that of the earth from the sun being known, its † geocentric place may likewise, at any time, be discovered. Its proportional distance is thus found.

D The figure as before. Suppose the earth to have moved from E to C, in a given time. From the time may be found the quantity of the angle E S C; and in the same time Jupiter will have moved to B, the angle J S B being also known from its proportion to his whole periodical revolution. Subtract the angle J S B from the angle J S C, and the remainder will be the angle B S C. By observation find the angle B C S, or Jupiter's elongation from the sun. In the triangle C B S, the sum of the two angles B C S and B S C being taken from 180 degrees, leaves the angle C B S. Then, by plain trigonometry,

E As the sine of the angle of the earth's elongation, when viewed from Jupiter C B S
Is to the sine of the angle of Jupiter's elongation, when viewed from the earth B C S,
So is the earth's distance from the sun C S
To Jupiter's distance from the sun B S.

F The angle of the earth's elongation, when viewed from Jupiter, is called Jupiter's annual parallax,

* Viewed from the sun as a center.

† Viewed from the earth as a center.

and

and is always equal to the difference between its heliocentric and geocentric place in the ecliptic, as a little conſideration will ſhew.

By ſimilar obſervations on the other ſuperior G planets, it is found that their apparent motions are attended with circumſtances of the ſame nature as thoſe of Jupiter. The ſame conſequences muſt therefore follow reſpecting their orbits, periods, and other affections.

Thus far we have ſpoken of the appearances of H the planets, as if their revolutions were performed in circular orbits, in the center of which the ſun was ſuppoſed to be placed. But this is not the caſe. Conjunctions, oppoſitions, ſimilar elongations, or other mutual ſituations of the planets, do not return again in exactly the ſame time, and their diſtances from the ſun are found to be greater or leſs in different parts of their orbits, their angular velocities being always greater when the diſtances are leſs. Thus, by the increaſed diameter of the I ſun during the winter half-year, we find that the earth's diſtance is diminiſhed; and that its velocity is increaſed, is evinced from the apparent motion of the ſun, by which it paſſes through the winter half-circle of the ecliptic in near eight days leſs than it employs to deſcribe the ſummer-half. By a variety of obſervations of elongation or parallax, the relative or proportional diſtances of the planets from the ſun, and their velocities are found for every heliocentric poſition. Whence K they are proved to revolve in elliptical orbits, the

ſun

sun being placed in one of the foci; and their velocities are such, that a radius drawn from the sun to the planet, and supposed to move with it, describes equal areas in equal times.

L The distance between the center S (fig. 41) and one of the foci C of an elliptical orbit, is called its eccentricity. The two extreme points of the transverse or longest diameter, L and U, are called the apsides. If the focus about which the equal areas are described be at C, the point L nearest that focus is called the lower apsis, and U is called the upper apsis; the diameter U L being called the line of the apsides. But it is more common to say, that a planet is in its perihelium when at L, and in its aphelium when at U. When the earth is in its perihelium, the sun is said to be in its perigee, and when the earth is in its aphelium, the sun is said to be in its apogee.

M From the generation of the ellipsis (97, T) it is evident, that the whole length of the string is equal to the transverse diameter: because, when the point A is at U, the return of the string from U to the nearest focus C, is exactly equal to the part between L and the other focus C, where there is no string. Let OH be the conjugate, or shortest diameter of the ellipsis, and CO will be equal to half the string, or half the transverse diameter. Suppose now a planet to revolve in the ellipsis about the focus C nearest L. Then CL will be its nearest distance, CU its greatest distance, and CO its mean distance. For, CO equal to SL, exceeds

ceeds the leaſt diſtance CL by the quantity CS, which is juſt as much as it falls ſhort of CU, the greateſt diſtance. If therefore the mean diſtance CG, and the eccentricity CS of a planet be given, its orbit may be deſcribed: becauſe the greateſt diſtance is equal to the ſum of the eccentricity and the mean diſtance; and the leaſt diſtance is equal to their difference.

The eccentricities of the planets are ſo ſmall, n
that their orbits approach nearly to circles.

If the plane of the earth's orbit were extended o
indefinitely every way, it would mark that circle in the heavens which is called the ecliptic, or ſun's path. If the orbit of any other planet be ſituated in this plane, it will always be ſeen in the ecliptic, whether viewed from the earth or the ſun. But if the plane of the planet's orbit be obliquely ſituated with reſpect to that of the ecliptic, it will interſect it in a line paſſing through the center of the ſun, and the planet will never be ſeen in the ecliptic but when in the points of interſection. Theſe oppoſite points of the ecliptic are called the nodes, and the line of interſection is called the line of the nodes. When a planet croſſes the ecliptic from ſouth to north, the node is termed the aſcending node; and when it croſſes from north to the ſouthward, the node is termed the deſcending node.

The orbits of all the planets are inclined to the p
ecliptic in ſmall angles.

CHAP.

CHAP. V.

OF THE AFFECTIONS OF THE PLANETS.

Q By supposing ourselves in the place of one of the ancients who discovered the order of the planetary system, we have displayed in a cursory manner some of the most obvious phenomena, and pointed out their natural consequences. What has been said is sufficient to shew to those who are totally unacquainted with the subject, that the doctrine of a system of bodies revolving round the sun is not merely ideal, but founded on the most natural deduction from the celestial appearances. For the processes by which the planets places are determined in elliptical orbits, we refer the reader to treatises written expressly on the subject, and in the mean time proceed to note several of those affections of the heavenly bodies, as determined by the accurate observations of modern times.

R Seven planets, Mercury, Venus, the Earth, Mars, Jupiter, Saturn, and the Georgium Sidus*, revolve about the sun in orbits included within each other, in the order here used in mentioning their names, Mercury being nearest the Sun.

* This planet was discovered in the year 1781, by William Herschel, Esq. a native of Hanover.

Sun. These are called primary planets, besides which, there are fourteen which are called secondary planets, Moons or Satellites. The secondary planets respect the primary planets, performing their revolutions about them, but are at the same time carried round the Sun in the orbit of the primary. Saturn is attended by seven Moons, Jupiter by four, the Georgium Sidus by two, and the Earth by one, all which, except the last, are invisible to us, by reason of their smallness and distance, unless telescopes be made use of. Without this instrument, it would likewise be impossible to ascertain the apparent diameters of any of the celestial bodies, the Sun and Moon excepted. The following table exhibits some of the affections of the primary planets.

Anno 1784.	Mercury.	Venus.	Earth.
Greatest possible elongation of inferior and parallax of superior planets, -	28°. 20′	47°. 48′	* *
Proportional mean distances from the sun, - -	38710	72333	100000
Periodical revolutions,	87d. 23 h. 15⅔ m.	224 d. 16 h. 49⅓ m.	365 d. 6 h. 9¼ m.
Diurnal rotations, - -	Unknown.	23 h. 22. m.	23 h. 56 m. 04 s.
Inclinations of their orbits to the ecliptic, - -	7° 0′	3° 23 ⅓′	* * *
Eccentricities, - -	7960	510	6180
Place of the ascending node, - -	1 s. 15 deg. 46¾ m.	2 s. 14 d. 44 m.	* * *
Place of the aphelium,	8 s. 14 d. 13 m.	10 s. 9 d. 38 m.	9 s. 9 d. 15½ m.
Proportion of axis to the equatorial diameter, - -	Unknown.	Unknown.	2289 to 2300
Greatest apparent diameters, - -	11″	58″	*
Diameters, if seen at the sun's mean distance, or proportional diameters,	7″	18″. 7	17″. 3
Diameters in geographical miles; that of the sun being 762490, -	2782	6637	6875¼
Mean distances from the sun in semidiameters of the earth,	9210	17210	23799
Mean distances from the sun in geographical miles, -	31671900	59181700	81818400
Proportions of light, -	668	191	100

Mars.	Jupiter.	Saturn.	Georgium Sidus, 1782.
47°. 24′	11°. 51′	6°. 29′	3°. 4¼′
152369	520098	953937	1903421
686 d. 23 h. 30¾ m.	4332 d. 8 h. 51½ m.	10761 d. 14 h. 36¼ m.	30445 d. 18 h.
24 h. 39 m. 22 s.	9 h. 56 m.	10 h. 16 m. 0,4 s.	Unknown.
1°. 51′.	1° 19¼′	2°. 30⅓′	46′ 12″
14218	25277	53163	4759
1 s. 17 d. 59 m.	3 s. 8 d. 50 m.	3 s. 21 d. 48⅓ m.	3 s. 13 d. 1 m.
5 s. 2 d. 6¼ m.	6 s. 10 d. 57½ m	9 s. 0 d. 45½ m.	11 s. 23 d. 23 m.
1272 to 1355	13 to 14	10 to 11.	Unknown.
25″	46″	20″	4″
9″ 1	193″. 7	171″. 7	74″. 5
3617	76982	68238	34217
36262	123778	227028	453000
124666000	425536000	780395100	1557350000
43	3 7	1. 1	0. 276

CHAP. VI.

OF PARALLAXES, AND OF THE TRANSIT OF VENUS.

S ONE of the moſt uſual methods of meaſuring inacceſſible diſtances, is by means of two ſtations whoſe diſtance from each other is know, and the angles formed at each ſtation between lines ſuppoſed to be drawn from the diſtant object to them, and the line that joins the ſtations to each other. Thus the diſtance between A and B (fig. 48) being known, as likewiſe the angles C A B and C B A, the diſtance A C or B C may be readily found by plane trigonometry.

T Suppoſe the object C, when viewed from B, (fig. 48) to coincide with another object S, which is at a diſtance indefinitely great; then the object C will not appear to coincide with S, when viewed from A. For S, on account of its great diſtance, will be ſeen in the line A S, parallel to B S; and C will be ſeen in the line A C, the angle S A C being the difference between the apparent places from A and B. This angle, becauſe of the parallels A S and B S, will be always equal to the angle A C B, and is by aſtronomers called the parallax. It is uſually diſtinguiſhed by ſome appellation relative to the nature of the line A B: for inſtance, it is called the annual parallax, when A B is the radius of the annual orbit; the diurnal parallax, when A B is the ſemidiameter of the earth, &c.

It

It is ſcarcely neceſſary to obſerve, that the longer u
the line A B is in proportion to the diſtance of C, the greater the angle A C B, and that in moſt practical caſes, the greater the angle the leſs is the diſtance affected by any ſmall error. It is therefore requiſite that the baſe A B be as large as poſſible or convenient.

The diſtances of the planets were found by tri- v
gonometry, the diſtance of the earth from the ſun being aſſumed as a baſe. But as that baſe cannot actually be meaſured, the diſtances are only proportional or relative, the baſe being ſuppoſed to be divided into 100000 equal parts; but whether thoſe parts be miles, leagues, or anſwer to any other denomination of length, was not determined. The real diſtances muſt be diſcovered by a parallax whoſe baſe is known.

The diameter of the earth is in general uſed as w
the baſe for determining the diſtances of celeſtial objects by their parallax, which parallax is found as follows.

Let A O B (fig. 49) repreſent the earth, C its cen- x
ter, and Z the zenith or point in the heavens, ſituated perpendicularly over the point O at its ſurface. Then C H will be the rational horizon, and O K the ſenſible horizon. Suppoſe a ſpectator at C views a celeſtial object at Z, the revolution of the earth will cauſe it to move apparently through the quadrant Z H in ſix hours, at the end of which time he will ſee it in the horizon at H. But to a ſpectator at O it will ap-

pear in the horizon when at K, after passing through the apparent quadrant or right angle Z O K, in a time as much less than six hours as the arc Z K is less than Z H, or 90 degrees. Hence the time of an object's passing between the zenith and sensible horizon being known, the angle O K C, or horizontal parallax may be found. For, as six hours is to 90 degrees, so is the time observed, to the arc Z K, which being taken from 90 degrees, leaves the arc K H measuring the angle K C H, which is equal to O K C, or the horizontal parallax.

Y The horizontal parallax being discovered, the distance of the object follows by this analogy; in the triangle O K C.

As the horizontal parallax, sine	- -		O K C
Is to the earth's semidiameter	- - -		O C
So is radius - - -	sine	-	90°
To the distance - . -		-	C K

The fixed stars have no parallax, either horizontal or even annual, whence it follows, that their distances are beyond all comparison greater than that of the earth from the sun.

Z It is obviously unnecessary in observations of parallax to wait till the object has described the whole apparent quadrant Z K (fig. 49): for, when it is arrived at E, the angle Z C E may be known from the time, and Z O E from observation, and their difference will be the angle of parallax O E C: so that in the triangle O E C are given two angles, and the side O C, from whence the other parts are easily found.

Obſervations of parallax are conveniently made B by the help of the fixed ſtars. Thus, if the object E when at Z be ſeen in a given poſition with reſpect to a fixed ſtar, it will continue to have the ſame poſition when arrived at E, provided the ſpectator be at C; but if the ſpectator be at O it will be ſeen depreſſed below its former poſition by the quantity of the parallax D O E, becauſe the ſtar has no parallax, but is ſeen in the ſame apparent place either from O or C.

The Sun's parallax is ſo exceedingly ſmall, that C the beſt inſtruments in the hands of the moſt ſkilful obſervers, have ſcarcely effected more than to ſhew that it has one. To remedy this, the horizontal D parallaxes of the nearer planets have been attempted, particularly of Mars, when in oppoſition to the Sun, this planet being then as near again to the Earth as the Sun is, and has therefore a parallax twice as great. But as this parallax is not found to exceed half a minute of a degree, the unavoidable uncertainty of obſervation, and other cauſes, render it not ſufficiently exact to determine the Sun's diſtance within a 30th part of the whole. It is eaſy to comprehend how the Sun's diſtance may be found when the diſtance of Mars, from the Earth, in oppoſition, is known. Thus, if S (fig. 50) be the Sun, E the Earth, and M Mars, in oppoſition, then E M will be the diſtance of Mars from the Earth, and alſo the difference between M S and E S, or the reſpective diſtances of Mars and the Earth from the

Sun. The proportional diſtances are known. Therefore,

As the difference between the proportional diſtances of Mars and the Earth from the Sun,

Is to the proportional diſtance of the Earth from the Sun;

So is the diſtance between the Earth and Mars in oppoſition, or the difference between their real diſtances from the Sun,

To the Earth's real diſtance from the Sun.

E Several other methods were deviſed by the ancients for diſcovering the Sun's parallax, which, though they ſhew the ſagacity and penetration of their inventors, are leſs ſufficient for the purpoſe than the foregoing. We ſhall therefore omit mentioning them, and give a ſhort explanation of that for which we are indebted to the great Dr. Halley, by which the diſtance of the Sun is determined with greater accuracy than by any other method.

F The planet Venus, as has been ſhewn, paſſes the Sun twice in revolving from any poſition of elongation to the ſame poſition again (105, c). At thoſe times this planet is ſaid to be in conjunction with the Sun.

G When the planet Venus is ſituated in a line between the Sun and the Earth, it is ſaid to be in its inferior conjunction; and when it is in the oppoſite part of its orbit, the Sun being in a line between it and the Earth, it is ſaid to be in its ſuperior conjunction. If the orbits of the Earth and Venus were

were in the ſame plane, it is evident that Venus would paſs behind the Sun with a direct motion every ſuperior conjunction, and would paſs over its * diſc, or before it, with a retrograde motion every inferior conjunction. But as Venus's orbit is inclined to the ecliptic in an angle of about 3½ degrees, this planet will, in general, paſs to the northward or ſouthward of the ſun, and will only be viſible on its diſc when the inferior conjunction happens at or near one of the nodes. This happens but once (or ſometimes twice at an interval of about 8 years) in more than 120 years.

To ſhew how this tranſit is applied to the pur- H
poſe of finding the Sun's diſtance, we ſhall paſs over thoſe elements that enter into the computation previous or ſubſequent to actual obſervation, and ſhall only explain the general principles on which the method is founded.

Let S (fig. 51) repreſent the Sun, E the Earth, I
V, U, W, the planet Venus in different poſitions, the arc L N a part of the Earth's orbit, and the arc O M a part of the orbit of Venus. Then, becauſe the angular velocities of Venus and the Earth are known, as alſo their proportional diſtances, it will be eaſy to compute the time Venus will employ in paſſing through the arc V W, which, when viewed from the Earth, is equal to the known diameter (or chord) of the Sun C D; the heliocentric value or length of the arc V W may likewiſe be readily found. Suppoſe then an obſerver at A on the

* Surface.

Earth's ſurface to view the planet Venus at v, it will appear juſt entered within the Sun's diſc at c, and paſſing in the arc v w, will appear to deſcribe the line c D, arriving at D at the end of the computed time. But during this time the obſerver will, by the Earth's diurnal revolution, be carried from A towards P; and arriving at P at the ſame inſtant that Venus arrives at u, will behold the tranſit juſt finiſhing at D: conſequently it will be of a duration proportionally as much ſhorter than the computed time, as the heliocentric arc v u is ſhorter than v w. The arc v w is known by computation, therefore, ſince Venus's motion may in very ſmall arcs be reckoned uniform,

As the computed time
Is to the computed arc v w,
So is the obſerved time
To the arc - v u;

which being taken from v w, leaves the arc u w, that ſubtends the angle u D v. This laſt angle is the parallax of the baſe A P; and the baſe A P is found by the analogy

As one day or 24 hours
Is to the circumference of the earth (or parallel of latitude)
So is the obſerved time -
To the arc A P, whoſe chord is the baſe.

K But becauſe the minuteſt errors in a buſineſs of this nature are of very great conſequence, and becauſe

cáuſe the length of the arc v w, depending on the Sun's diameter, can ſcarcely be obtained by calculation to that extreme degree of exactneſs, which is requiſite, it is adviſeable to take another obſervation on a place ſo ſituated on the earth, that the obſerver being carried in a direction apparently contrary to the former, the errors may counteract each other.

Let the repreſentations be as in the laſt figure. L
If the ſun have declination at the time of the tranſit, B (fig. 52) will repreſent the pole towards which the ſun declines. The obſerver at A, if at reſt, would behold the tranſit during the time Venus paſſes from V to W, but being by the Earth's diurnal revolution carried from A through the arc A E P to P, and arriving at P at the inſtant in which Venus arrives at U, he will perceive the tranſit juſt finiſhing at D; conſequently its duration will be as much longer than the computed time as the heliocentric arc V U is longer than V W. V U being found by the before mentioned analogy, the difference between V U and V W is W U, or the parallax of A P, as before.

Now, in theſe two caſes, a ſimilar error will M
have a contrary effect in the firſt to that which it has in the latter. For if by any error, the computed arc V W (fig. 51) be taken too large, the arc U W, and conſequently the parallax will come out too great. But in the latter obſervation, if the computed arc V W (fig. 52) be taken too large, the arc W U, and conſequently the parallax will come out too little. Therefore the mean between

two fuch obfervations will be much more to be depended on than either fingly.

N By obfervations on the tranfits of Venus over the Sun in the years 1761 and 1769, the Sun's mean parallax was found to be $8\frac{2}{3}$ feconds, and hence the Sun's diftance is deduced to be very near 11900 diameters of the Earth, or 81818400* geographical miles.

The laft three articles in Chap. V. concerning the affections of the planets are deduced from this diftance; for,

As the proportional diftance of the earth
Is to its real diftance,
So is the proportional diftance of any other planet
To its real diftance.

* A geographical mile is $\frac{1}{60}$ part of a degree of the earth.

CHAP.

CHAP. VII.

OF THE SECONDARY PLANETS.

THE fecondary planets, as was before obferved, are ten in number, five of which defcribe orbits about the planet Saturn, four about Jupiter, and one accompanies the Earth. The fecondaries of Saturn and Jupiter are obferved by the telefcope, and by their motions in elongation to the eaftward or weftward of their primaries is obtained the knowledge of their diftances and periodical times, in the fame manner as has been already fhewn and explained in the planet Venus. Saturn is likewife attended by a phenomenon, which to us appears to be a large broad ring, of no vifible thicknefs. Its breadth is equal to its diftance from the body of the planet, and its diameter is to that of Saturn as 9 to 4. The moft probable conjecture is, that it confifts of a vaft number of fatellites, which revolve in, and enlighten that region.

Of the two moons of the Georgium Sidus, the periodical times: Firft, 8^{d} 17^{h} 1^{m} 19^{s}; Second, 13^{d} 11^{h} 5^{m} $1\frac{1}{2}^{s}$. Diftances: Firft, 33″; Second, 44″ 23. —Of Saturn's feven moons, the periodical times: Firft, 1^{d} 21^{h} 19^{m}; Second, 2^{d} 17^{h} 41^{m}; Third, 4^{d} 13^{h} 47^{m}; Fourth, 15^{d} 22^{h} 41^{m}; Fifth, 79^{d} 22^{h} 41^{m}; Sixth, 1^{d} 8^{h} 53^{m} 9^{s}; Seventh, 0^{d} 22^{h} $40\frac{1}{3}^{m}$. —Diftances in femidiameters of the ring; Firft, $1\frac{95}{100}$; Second, $2\frac{5}{10}$; Third, $3\frac{52}{100}$; Fourth, $8\frac{9}{100}$; Fifth, $23\frac{71}{100}$; Sixth, 35.″058; Seventh, 27.″366 —Of Jupiter's four moons, the periodical times: Firft,

First, $1^d\ 18^h\ 27^m\ 33^s$; Second, $3^d\ 13^h\ 13^m\ 42^s$; Third, $7^d\ 3^h\ 42^m\ 33^s$; Fourth, $16^d\ 16^h\ 32^m\ 8^s$. Distances in semidiameters of Jupiter: First $5\frac{667}{1000}$; Second, $9\frac{17}{1000}$; Third, $14\frac{384}{1000}$; Fourth, $25\frac{299}{1000}$.

All the planets, both primary and secondary, receive their light from the Sun. This is evident, because that face only is enlightened which is turned towards that luminary, as may be more particularly seen in our Moon, a greater or less part of which is visible, according to the position in which we lie for viewing the illuminated face. The same varieties are seen in the planets Mars and Venus, not to mention the transits of Venus and Mercury over the Sun, at which time they appear as black unenlightened spots. The phases of Jupiter aud Saturn are always round and full, because the Earth is so near the Sun in respect to their distances, that their dark side can never be sensibly turned towards us; yet, that they are opaque, is evident from the disappearing of Jupiter's moons when they enter into its shadow. And though by reason of their vast distance the like obscurations of the satellites of Saturn cannot be observed, yet we can plainly see that the ring casts a shadow on its body: whence we may be certain of the opacity of both: for if the ring were not opaque it could cast no shadow, and if Saturn shone by any native light of his own, the interception of the Sun's light would cause no defect or shadow on his body. It is unnecessary to observe, that the Earth and its Moon are illuminated only on that part or side on which the Sun shines.

When

When one planet intercepts any part of the Sun's light from another, the planet from which the light is intercepted is said to be eclipsed, if it be a secondary. But if they are both primaries, the inferior planet is said to make a transit. When the moon intercepts the Sun's light from the Earth, it is usual to say the Sun is eclipsed, though, properly speaking, it is the Earth that is eclipsed. s

There are three ways in which the satellites of Jupiter or Saturn may disappear from an observer placed on the Earth. Thus, let S (fig. 53) represent the Sun, E the Earth in its orbit, J the planet Jupiter and its moons. Then the outermost satellite, for example, will disappear on the enlightened face of Jupiter when at its inferior conjunction M. It will also disappear at its superior conjunction N, being hid behind the body of the planet. And lastly it will disappear when at O, being eclipsed in passing through the shadow of Jupiter. T

From these considerations is obtained a good method of finding the parallax of the Earth's annual orbit. For which purpose the instant of the satellite's first disappearance behind the body of Jupiter must be carefully observed, as likewise the instant of its re-appearance: the middle instant will be that of the superior conjunction at N. In like manner, the middle-instant of the eclipse at O must be found. The time the satellite employs in passing through the arc N O will thus be known, and consequently the angle N J O. For, U

As the periodical time of the ſatellite
Is to the time of paſſing the arc N O,
So is the whole orbit of 360 degrees
To the angle N J O.

But the angle N J O is equal to the angle E J S, or the annual parallax.

v By the obſervations of theſe eclipſes, the diſcovery of the longitude on ſhore is eaſily obtained, but the violent motion of ſhips at ſea has hitherto prevented the uſe of teleſcopes on board proper for this purpoſe, though there are good reaſons to believe that this w difficulty is not inſurmountable. From theſe obſervations it alſo appears, that light is not propagated from luminous bodies in an inſtant, but paſſes through a given ſpace with an aſſignable velocity. This velocity is extremely great, for it paſſes through the whole diſtance between the Sun and the Earth in about eight minutes; that is to ſay, at the rate of one hundred and ſeventy thouſand miles in a ſecond of time. For the periodical times of the ſatellites being known, it is not difficult to determine the preciſe time of any of their eclipſes. But it is found neceſſary to make an allowance for the poſition of the Earth with reſpect to Jupiter, ſince the eclipſes happen ſooner when the Earth is at F, (fig. 53) in its orbit, than when at a greater diſtance, ſuppoſe at E; and as it is abſurd to ſuppoſe, that the poſition of the Earth ſhould ſenſibly and equally affect the periodical revolutions of bodies ſo vaſtly remote, and revolving in ſuch different periods, it is an opinion univerſally received,

and

and confirmed by other obſervations on the light of the fixed ſtars*, that the eclipſes happen later when the Earth is at E than when at F, becauſe the light muſt in the latter caſe paſs through a ſpace as much greater as the line J E exceeds J F.

CHAP. VIII.

OF THE MOON.

THAT the Moon revolves round the Earth, is proved from its apparent diameter, which continues at all times, and in all poſitions, nearly of the ſame magnitude, whence it may be eaſily inferred, that its diſtance from the Earth is nearly at all times the ſame. Its horizontal parallax, which at a medium is about 57, ſhews that it is very much nearer to us than the reſt of the celeſtial bodies.

The moſt remarkable appearance in the Moon is the continual change of figure to which it is ſubject. Sometimes it appears perfectly full or circular, at other times half-illuminated, and at other times more or leſs than half; changing through a very great variety of figures. Theſe changes being always the ſame at the ſame elongation from the Sun, are a proof that it receives its light from that luminary: for the Moon is enlightened only on the ſide that faces the Sun; and a greater or leſs quantity of that enlightened part is viſible to us, according to our

* The aberration of the fixed ſtars ariſing from the progreſſive motion of light, will be explained in Book II. Sect. I.

poſition.

poſition. This cannot be better illuſtrated than by an ivory ball, which being held in the Sun in various poſitions, will preſent a greater or leſs part of its illuminated ſide to the view of the obſerver. If it be held nearly in oppoſition, ſo that the eye of the obſerver may be almoſt immediately between it and the Sun, the greateſt part of the enlightened ſide will be ſeen. But if it be moved in a circular orbit towards the Sun, the viſible enlightened part will gradually decreaſe, and at laſt diſappear when the ball is held directly towards the Sun. Or, to apply the experiment more immediately to our preſent purpoſe; if the ball at any time, when the Sun and Moon are both viſible, be held directly between the eye of the obſerver and the Moon, that part of the ball on which the Sun ſhines will appear exactly of the ſame figure as the Moon itſelf.

z The Moon's path or orbit is inclined to the plane of the ecliptic, in an angle of about five degrees and
A a quarter. Its periodical revolution is performed in twenty-ſeven days, ſeven hours, forty-three minutes, eleven ſeconds and a half; but becauſe, during that time the Sun, by its apparent motion, advances conſiderably in the ecliptic, a ſpace of about two days and a quarter is required by the
B Moon to overtake it. When the Moon is as nearly in a line between the Earth and the Sun as the inclination of its orbit will allow, it is called the New
C Moon; and when the Earth is in like manner between the Moon and the Sun, the Moon is ſaid to

be

be full. The time between two ſucceeding full D
moons is called the ſynodical revolution, and exceeds the periodical revolution, for the reaſon already given, it being performed in twenty-nine days, twelve hours, forty-four minutes, and three ſeconds. If the new or full Moon happen near the E
node, an eclipſe takes place; at the new Moon, the Moon being interpoſed between the Sun and Earth, occaſions an eclipſe of the Sun; at the full, the Moon entering into the ſhadow of the Earth, is deprived of the Sun's light, the Earth being interpoſed between it and the Sun: which phenomenon is called a lunar eclipſe, or eclipſe of the Moon. At other times, that is, when the new or full Moon happens at a diſtance from the node, the Moon paſſes too far to the northward or ſouthward of the ecliptic, either to intercept the Sun's light from the Earth, or to enter the Earth's ſhadow, and conſequently no eclipſe happens.

It is determined from obſervations of angular F
velocity, parallax and apparent diameter, that the Moon revolves round the Earth in an elliptical orbit, in the focus of which the Earth is placed: and that its velocity is ſuch, that a radius joining its center with that of the Earth does very nearly deſcribe equal areas in equal times.

The line of the apſides, or principal diameter G
of the Moon's orbit, is not fixed or ſtationary, but revolves with an irregular or libratory motion from weſt to eaſt: compleating one revolution in almoſt nine years.

The

H The line of the nodes is alſo ſubject to a like irregular motion from eaſt to weſt; which is completed in almoſt nineteen years.

I The variation of the Moon's motion in any part of its orbit is the difference between its real motion and that which it would have had, provided it had deſcribed equal areas in equal times. This is governed chiefly by its elongation from the Sun. During the firſt quarter its velocity is diminiſhed; in the ſecond quarter, from the quadrature to the oppoſition or full Moon, it is increaſed; in the third quarter, from the oppoſition to the laſt quadrature, the velocity is again diminiſhed; and from that quadrature to the conjunction, its velocity is again increaſed. The quantity of angular motion loſt exceeds the quantity gained: therefore the whole periodical revolution is performed in a longer time than would have been employed if the Moon were ſubject to no ſuch variation, but deſcribed equal areas in equal times.

K This variation, and conſequently the retardation of the periodical time, is greater when the Earth is in the perihelium, and leſs when the Earth is in the aphelium: whence it comes to paſs, that all the Moon's revolutions are not equal, but are performed in leſs time in the latter ſituation than in the former.

L On all theſe, as well as other accounts, the determination of the Moon's place in the heavens for a given inſtant of time has ever been a pro-

 blem

blem of great difficulty, which till of late years has not been ſolved to any conſiderable degree of exactneſs. Within the laſt twenty years the commiſſioners, appointed by the Engliſh government for the diſcovery of the longitude, have particularly attended to this branch of aſtronomy, and by publiſhing almanacs in which the Moon's elongation from the Sun, and from certain fixed ſtars, is aſcertained for every three hours, have enabled navigators to determine the ſituation of ſhips at ſea in general within thirty miles of the truth. This is an advantage of ſingular uſe in long voyages, and is at preſent much uſed in the royal navy, and Eaſt India Company's ſhips.

CHAP. IX.

CONCERNING THE ECLIPSES OF THE SUN AND MOON.

WE have ſeen in what manner the periodical revolutions of the celeſtial bodies, together with the figure, magnitude, and poſition of their orbits, may be reſpectively determined by obſervations made on their apparent motions and ſituations. From the properties of the ellipſis, and the eſtabliſhed law of their velocities (121, K), or otherwiſe, more immediately from the conſideration of gravity (98, W, X), aſtronomical tables are computed, by which the places of the heavenly bodies may be found for any inſtant of time. The con-

ſtruction and uſe of theſe would lead us too far from the conciſe and exterior view of phenomena that our limits require: we ſhall therefore aſſume,
M as a thing granted of courſe, that the place of any celeſtial body may be found for any given inſtant of time.

N The eclipſes of the Sun and Moon are phenomena that command the attention even of the vulgar, who have always retained a ſuperſtitious veneration for the ſcience of aſtronomy, chiefly on account of the means it affords of foretelling events of this nature. And though in reality the knowledge required in calculating an eclipſe does not eſſentially differ from that employed in determining the time of the riſing and ſetting of the Sun or Moon, yet there is no doubt but a more particular attention to this ſubject will be acceptable to the reader.

O As the ſhadows of the Moon and Earth are the cauſes of eclipſes, it will be neceſſary firſt to determine the figure of thoſe ſhadows. Becauſe the Sun, the Earth, and the Moon are ſpherical bodies, it follows that the ſhadows of the two latter muſt be
P either conical or cylindrical; that is to ſay (fig. 54), if the Sun I K be leſs than the Earth C D, the ſhadow of the latter will be part of a cone, whoſe ſection is terminated by the lines C E, D F, and whoſe baſe is indefinitely diſtant: or, if the Sun A B be equal to the Earth C D, the ſhadow will be a cylinder between the lines C G, D H, whoſe baſe is indefinitely diſtant. In either caſe the ſhadows of the

Earth

Earth may conſequently fall upon and eclipſe the ſuperior planets, when in direct oppoſition to the Sun. But this never happens, and therefore the Sun is neither leſs than, nor equal to, the Earth, but greater. We know moreover, from the Sun's parallax (136, N), that it is much greater than the Earth, becauſe the Sun's diameter ſeen from the Earth is about 32 minutes, whereas the Earth's Q
diameter ſeen from the Sun is (126) only about 17 ſeconds, a quantity that may be regarded as inſenſible, or inconſiderable in many obſervations. And ſince the Sun exceeds the Earth in ſo high a R
proportion, it muſt of neceſſity be yet greater with S
regard to the Moon, becauſe this laſt is leſs than the Earth. Let A B (fig. 55) repreſent the Sun greater than the Earth C D. The rays of light A C, B D, paſſing from the extreme edges of the Sun, and in contact with the Earth on the ſame ſide, will afterwards meet or croſs in the point K. No part of the Sun's light will appear within the cone C K D, which is therefore the ſhadow in which an obſerver, being placed, would be totally deprived of the Sun. But there will be a partial ſhadow or penumbra between thoſe rays A D M, B C L, that paſs from the extreme edges of the Sun, and touch the oppoſite extremes of the Earth: that is to ſay, an obſerver between the lines C L and D M, but without the dark cone C K D, will ſee only a part of the Sun, the reſt being hidden by the interpoſition of the Earth: the quantity of the Sun thus obſcured will be greater, and the penumbra darker, the nearer the obſerver is

placed to the cone C K D. Laſtly, if the obſerver be ſituated beyond the vertex of the dark ſhadow K, between the lines K N, K O, formed by the continuation of the extreme rays, he will behold the exterior parts of the Sun forming a lucid ring, environing the Earth on all ſides.

T The angle C K D, at the vertex of the Earth's ſhadow, is equal to the difference between the diameter of the Sun, ſeen from the Earth or angle A C B, and the diameter of the Earth ſeen from the Sun, or angle C B D*. Or, if the Earth's apparent diameter from the Sun (147, Q) be rejected as inconſiderable, the angle of the ſhadow will be equal to the Sun's apparent diameter.

U The angle C I D, at the vertex of the penumbra, is equal to the ſum of the diameter of the Sun ſeen from the Earth, or angle A C B, and the diameter of the Earth ſeen from the Sun or angle C A D: or, if the Earth's apparent diameter from the Sun (147, Q) be rejected as inconſiderable, the angle of the penumbra is equal to the Sun's apparent diameter.

V The apparent diameter of any ſection E F, of the ſhadow, ſuppoſed to be viewed from the Earth, namely, the angle E D F, is equal to the exceſs of

* Euclid I. 32. is repeatedly uſed in what immediately follows: that is to ſay, in any triangle B C K, the outward angle A C B formed by prolonging one of its ſides, is equal to the ſum of the two inward oppoſite angles C K D, C B D: and conſequently, that one of the two laſt-named angles, or C K D, will be equal to the difference between the external angle A C B and the other interior oppoſite angle C B D.

the

the Earth's apparent diameter ſeen from the place of ſection, namely, the angle C E D, beyond the angle at the vertex of the ſhadow C K D: or, if the angle of the ſhadow (148, T) be taken as equal to the Sun's apparent diameter, the apparent diameter of any ſection of the ſhadow ſeen from the Earth will be equal to the difference between the apparent diameters of the Sun and Earth, as ſeen from the place of ſection, this laſt diameter being greateſt.

The apparent diameter of any ſection G H, of the penumbra, ſuppoſed to be ſeen from the Earth, namely, the angle G D H, is equal to the ſum of the Earth's apparent diameter ſeen from the place of ſection, namely, the angle C G D, added to the angle at the vertex of the penumbra C I D. Or, if the angle of the penumbra (148, U) be taken as equal to the Sun's apparent diameter, the apparent diameter of any ſection of the penumbra ſeen from the Earth will be equal to the ſum of the apparent diameters of the Sun and Earth, as ſeen from the place of ſection. W

Every thing that has been here ſhewn reſpecting the ſhadows of the Earth is true in like circumſtances of the Moon (147, S). X

To apply theſe obſervations to the facts, let A B (fig. 56) repreſent the Sun, C D the Earth, and I K or L the Moon in its orbit K M N; let the Moon be at I K, between the Sun and Earth; its total ſhadow may then entirely deprive a part of the Earth at O of the Sun's light, and its penumbra will cauſe a partial eclipſe of the Sun to the inhabi- Y

z tants between G and H. Again, ſuppoſe the Moon to be at L, and it will itſelf be eclipſed by the interpoſition of the Earth between it and the Sun. In lunar eclipſes, the Earth's penumbra is not attended to, becauſe its effects in obſcuring the Moon cannot be obſerved with preciſion by a ſpectator placed on the Earth.

A It has already been obſerved (143, E), that eclipſes can only happen when the Moon is near one of the nodes of its orbit. Let A B M (fig. 57) repreſent the Sun, viewed from the Earth, C D a portion of the ecliptic, or Sun's apparent path, and E F a part of the orbit of the Moon; which planet is repreſented at different times by the circles G, H; I. It is evident, that the eclipſe or obſcuration of the Sun entirely depends on the poſition of the node N, and the angle of inclination F N D. If the angle of inclination remain unaltered while the node N is very remote from the center K of the Sun, the points K and L may be farther apart than to permit any occultation or apparent contact; and it is clear, that an enlargement of the angle F N D may produce the ſame effect: on the contrary, an approach or coincidence of N with K, or a diminution of the angle F N D may cauſe an eclipſe, the quantity of obſcuration in which will be ſo much greater, as theſe circumſtances are more prevalent.

B The Sun's place K in the ecliptic (146, M) being known from tables, together with the inclination of the Moon's orbit, the place of the node, and of the Moon itſelf, as likewiſe the apparent diameters

of the luminaries reſpectively, it will be eaſy to find the velocity of the Moon in elongation, and conſequently the beginning, middle, end, quantity of obſcuration, and other requiſites concerning the eclipſe. If the computation be made from the ta- C
bular places of the heavenly bodies, the reſult will give the eclipſe as ſeen from the center of the Earth, becauſe, in all tables where the Earth is ſpoken of, that center is meant, except otherwiſe mentioned. But it is required to determine the particulars of the D
eclipſe for a given place on the Earth's ſurface, and this includes the conſideration of parallax. The Sun's parallax being very minute (136, N) may in this, and moſt other caſes, be rejected: but the Moon's parallax is ſo great, that it is at leaſt of as much conſequence as any other element whatſoever. For, on this account, the Moon's apparent path, as ſeen from the ſurface of the Earth, is ſo different from that which it would have when beheld from the center, that the ſame conjunction which gives a total eclipſe at one place ſhall not occaſion the ſmalleſt obſcuration of the Sun when beheld at the ſame inſtant from another part of the Earth.

This method of computing a ſolar eclipſe is very E
operoſe. For the Moon's parallax at any time paſt or to come cannot be had without finding its altitude by ſpherical trigonometry, and other computations muſt then be made to deduce the apparent poſitions of the Sun and Moon with their relative velocity, and ſo forth. And becauſe the altitude of the Moon is continually changing, it is

neceſſary to repeat the computations of parallax from time to time. To render this buſineſs leſs tedious, it has been found expedient to conſider the phenomena of ſolar eclipſes as they would appear to an obſerver placed on the Moon. Let A B (fig. 58) repreſent the Earth, ſeen from the Moon, under an angle of about 1° 54′, or double the Moon's horizontal parallax (141, x). the line C D a portion of the oppoſite part of the Moon's orbit in which the Earth is ſeen, the circles G, H, I, ſhadows of the Moon, that on account of their always being diametrically oppoſite the Sun, will be found to ſenſe in the ecliptic, eſpecially when the Moon is near the node. The path F E of the ſhadow will therefore make an angle F N D with the line C D, equal to the inclination of the Moon's orbit, and the interſection N will be as far diſtant from the center of the Earth as the node is heliocentrically from the center of the Moon. Now the motion of the Earth, in the line C D, is equivalent to the Moon's apparent motion in its orbit ſeen from the Earth, and the motion of the ſhadow is equivalent to the Sun's motion in the ecliptic. Conſequently, the center of the Earth and ſhadow of the Moon may be projected as ſeen from the Moon. The diameter of the dark ſhadow, K, L, or M, ſeen from the Moon, will be equal to the exceſs of the Moon's apparent diameter beyond that of the Sun, when both are ſeen from the Earth (148, v. 149, x), the Moon's apparent diameter being greateſt, but if it

it be the leſs of the two, the ſhadow will not reach the Earth. The diameter of the penumbra, G, H, or I, ſeen from the Moon, will be equal to the ſum of the apparent diameters of the Sun and Moon, ſeen from the Earth (149, W, X). With theſe data the eclipſe may be conſtructed univerſally.

But in conſtructing the eclipſe for a particular place, the rotation of the Earth on its axis muſt be brought into conſideration. For while the ſhadow paſſes over the Earth's diſc, a given place P will be carried round in its parallel of latitude, and may likewiſe be marked in the projection for any inſtant of time. When the place enters the penumbra, the eclipſe will begin there; when the place and the center of the ſhadow are the neareſt, the obſcuration will be greateſt, and when the penumbra leaves the place, the eclipſe will end. If there be a dark ſhadow, and it paſſes over the place, the eclipſe will be total. If there be no dark ſhadow, and the place ſhould paſs within the penumbra to a depth exceeding the Moon's apparent diameter, the Sun will be ſeen environing the Moon on all ſides; whence the eclipſe is ſaid to be annular (148, S). And, in general, in any ſolar eclipſe that is not annular, the diſtance of the place within the penumbra will meaſure the greateſt ſection or part of a diameter of the Sun obſcured at that inſtant, and the line joining the cuſps, or angular termination of the apparent part of the Sun, will be at right angles to the meaſuring line or diameter,

meter, of which the meaſuring line repreſents a part.

H We are now to conſider an eclipſe of the Moon. It is evident, that the difference in the phenomena of a ſolar eclipſe would not take place if the parallax of each luminary were the ſame; becauſe, whatever mutation of place the parallax might occaſion in the one, the ſame would be produced in the other, and they would neither approach nor recede from each other on that account. Now the ſection of the Earth's ſhadow paſſed through by the Moon in a lunar eclipſe, being at the ſame diſtance from the Earth as the Moon itſelf, muſt be ſubject to the ſame parallax at equal altitudes; and ſince the individual points of immerſion, emerſion, or other periods of the eclipſe muſt in the ſhadow have the ſame altitudes the parts of the Moon they, as it were, lie on and obſcure, the effects of parallax muſt be the ſame on both. Rejecting therefore the conſideration of parallax, the Earth's ſhadow A B (fig. 59) may be taken to occupy a place in the heavens diametrically oppoſite the Sun, and having an equal and ſimilar motion to the apparent motion of that luminary, its apparent diameter, ſeen from the Earth, will be equal to the difference between the apparent diameters of the Earth and Sun, as ſeen from the Moon (148, v). Or it will be equal to twice the horizontal parallax of the Moon diminiſhed by the ſubtraction of the Sun's apparent diameter. And if the inclination of the orbit of the Moon be found, there will be a certain diſtance

diſtance of the node N from the center of the ſhadow
C, that will require the Moon near the oppoſition
to paſs through the Earth's ſhadow, and be con-
ſequently eclipſed. From the greater or leſs di-
ſtance of the node N, or M, it will be determined
whether the eclipſe will be partial or total; and
from the reſpective places, the quantity and direc-
tion of the relative velocity, together with the ap-
parent magnitudes of the ſhadow and the Moon,
all the particulars of the eclipſe may be known
without difficulty.

It may with great reaſon be demanded, how it I
happens that the Moon, which is affirmed to emit
no light of itſelf, but only by reflection of the Sun,
is nevertheleſs ſufficiently luminous, even in the
very middle of a total eclipſe, to be diſtinctly ſeen
of a duſky reddiſh color. The Earth's atmoſphere,
or body of air that ſurrounds it, is the cauſe of
this phenomenon. In fact, the ſhadow of the K
Earth itſelf never extends ſo far as the Moon's or-
bit, though the ſhadow occaſioned by the diſper-
ſion or reflection of the light that falls on the at-
moſphere may, with a very ſmall allowance, be
taken for the ſhadow which the Earth would have
had if the light had paſſed cloſe by it without in-
terruption. We cannot with regularity explain the
refraction of light in this place. It will therefore
be ſufficient to obſerve, that in the event now under
conſideration the Sun's light falling obliquely on
the atmoſphere, is bent or turned out of its courſe
ſo as to converge ſooner to a point, than it would
otherwiſe

otherwiſe have done; the ſpherical atmoſphere performing, in ſome meaſure, the office of a large convex lens, or burning-glaſs. The more obliquely the rays fall, the greater is their deviation from their original courſe; and thoſe rays that paſs cloſe to the Earth are found, by obſervations on the ſetting Sun and other heavenly bodies, to ſuffer a refraction of about 33 minutes of meaſure. The laws of optics, hereafter to be explained, require that they ſhould undergo an equal refraction in paſſing out through the oppoſite part of the at-
L moſphere. Each exterior ray of the real ſhadow will therefore paſs 66 minutes within the rays that would have formed the cone C K D, fig. 55. and conſequently, the angle at the vertex of the cone will be 132 minutes, or 2°. 12 greater than it would have been; that is, it will be equal to the diameter of the Sun 32 (148, T), added to
M 2°. 12, which gives 2°. 44'. Hence the axis of the cone, or length of the ſhadow, is found to be no more than 42 ſemidiameters of the Earth; whereas the radius of the Moon's orbit, or mean diſtance of the Moon, is about 60 ſemidiameters
N of the Earth. In the ſpace between the penumbra and the Earth's real ſhadow it is much darker than the penumbra, though that ſpace is illuminated by the rays of the Sun, which are variouſly refracted, according to the denſity of the air they paſs through. Many rays are reflected back, and the rays that go forward are ſuch whoſe nature does not admit of their being eaſily reflected. We are to ſhew in future

future that these are the red, orange, and yellow. Hence it is that the Moon in an eclipse appears red; and a spectator on the Moon would, after losing sight of the Sun, behold the Earth environed with a narrow luminous edge of bright red light, shaded off with yellow on the outside.

Since the Earth, when beheld from the Moon, must always appear in the part of the heavens immediately opposite the Moon's apparent place as seen from the Earth, the enlightened side of the Earth will have the same figure, when seen from the Moon, as the dark side of the Moon would exhibit if it could be seen at the same instant from the Earth. Thus, when the Moon is invisible, or near the conjunction, the Earth is in opposition, and presents a full luminous face to the Moon; and on the contrary, when the Moon is at the full, or opposite the Sun, it must be on the dark side of the Earth, which consequently then becomes invisible. Near the beginning of the first or end of the last quarter, the dark side of the Moon is rendered visible by the full Earth shining on it, but is scarcely so luminous as the Moon when eclipsed. The Earth's disc, seen from the Moon, is about thirteen times that of the Moon seen from the Earth. If the Earth reflected as great a part of the light that falls on it as the Moon does, its light at the Moon would exceed the moon-light with us in that ratio. This, however, is not probable, though it may fairly be supposed that it is three or four times as great. But

we have already obſerved that the Earth's atmoſphere, in a lunar eclipſe, illuminates the Moon rather more than this. Whence it follows, that the narrow ring of light encircling the Earth, when ſeen from the Moon during an eclipſe, gives a light far exceeding our moon-light at the full, or even that of the Earth's full face ſhining on the Moon: and as the ſurface of the ring, by computation, can hardly equal the one hundredth part of the Earth's diſk, or the one eighth part of the Moon's diſk, its brightneſs will be more than twenty-four times that of the Moon. It muſt conſequently be very luminous and dazzling.

Notwithſtanding this, there have been eclipſes of the Moon, when in that part of its orbit near the Earth, in which that luminary entirely diſappeared. But theſe obſervations are very rare.

CHAP.

CHAP. X.

OF COMETS; AND OF THE PROPORTION OF LIGHT AND HEAT ON THE PLANETS.

Besides the ſeven primary planets already enumerated, and their moons or attendants, there are other bodies that revolve round the Sun, and claim peculiar diſtinction on ſeveral accounts. Theſe are called Comets, and appear occaſionally in every part of the heavens; their motions being performed in very long ellipſes, whoſe lower focus is in or near the Sun. By obſervations of parallax it is found, that at their firſt appearance they are nearer to us than Jupiter; whence it is concluded, that they are moſt commonly leſs than that planet; for if they were as large as Saturn, they would be ſeen as far off.

When a comet arrives within a certain diſtance of the Sun, it emits a fume or vapour, which is called its tail. This ſhews that they contain a portion of matter conſiderably more rare and volatile than any on the Earth; for the tail begins to appear while they are yet in a higher, and conſequently colder region than Mars. The tail is always directed to that part of the heavens which is directly

directly or nearly oppoſite to the Sun; and is greater after the comet has paſt its perihelium, than during its approach towards it; being greateſt of all at the time when it has juſt paſt the perihelium.

u That part of a comet's orbit which comes under our obſervation is ſo ſmall in proportion to the whole, that in moſt it does not differ from a parabola, by quantities that obſervation can diſtinguiſh: for which reaſon the dimenſions of their orbits and periodical times cannot be determined with any degree of preciſion from a ſingle appear-
v ance. But from the re-appearance of comets after long intervals of time in the ſame region of the heavens, and moving in the ſame curve, it is decided that they revolve about the Sun in very long or eccentric ellipſes; being governed throughout by the ſame law of deſcribing equal areas in equal times, which is found to take place in the inferior part of their orbits. The comet that appeared in the year 1661 was ſeen before in the ſame orbit, and under the ſame circumſtances in the year 1532: which ſhews its period to be 129 years. So likewiſe, the comet that appeared in the years 1456, 1531, 1607, 1682, and 1759, is determined to revolve in a period of about ſeventy-ſix years. And that very remarkable comet which was obſerved in the year 1680, is ſhewn to be the ſame with that which appeared in the year 1106; its period being 575 years.

The

The number of comets is very much greater w
than that of the planets which move in the vicinity of the Sun. From the reports of former historians, as well as from the observations of late years, it is ascertained, that more than four hundred and fifty have been seen previous to the year 1771: and when the attention of astronomers was called to this object by the expectation of the return of the comet of 1759, no fewer than seven were observed in the course of seven years. From this circumstance, and the probability that all the comets recorded in ancient authors were of considerable apparent magnitude, while the smaller were overlooked, it is reasonable to conclude, that the number of comets is considerably beyond any estimation that might be made from the observations we now possess. But the number of comets whose x
orbits are settled with sufficient accuracy to ascertain their identity when they may appear again is no more than fifty-nine, reckoning as late as the year 1771. The orbits of most of these are in- y
clined to the plane of the ecliptic in large angles, and the greater number of them approached nearer the Sun than the Earth ever does. Their motions z
in the heavens are not all in the order of the signs, or direct, like those of the planets: but the number whose motion is retrograde is nearly equal to that of those whose motion is direct.

It is not necessary in this work, to enter fully A
into the consideration of final causes; more particularly as the subject has embarrassed the greatest

metaphyficians, and may with juftice be faid to be too extended for the human powers. In every thing we fee, the phenomena confidered fingly, are neceffary confequences of certain general laws, to which the univerfe appears to be fubjected, but when they are confidered in a collective view, a certain relation, or fitnefs for producing fome general effects, is feen, which by no means depends on the fame laws, and by analogy is referred to the operation of an intelligent agent. To illuftrate this by an example, of the fimpleft kind, we may obferve, that in the well-known inftrument called fciffars, it follows neceffarily from the laws of motion, already explained in the mechanical powers (60, G. 67, H), that the blades will cut or divide certain fubftances expofed to their action; but if we confider the various circumftances that co-operate in producing this effect, we muft difclaim all reafoning from analogy before we can refolve their connection into an effect of thofe laws, without fuppofing the agency of an intelligent being as the caufe of their union, and concluding that it was intended they fhould jointly concur in one purpofe. It is to this being that we refer, in order to decide, why the fharp edges were made on the inner, rather than on the outer, part of each blade: why the other extremities have annular terminations: why the inftrument is made of fteel rather than lead; and fo forth. The purpofes or motives which determine the actions of intelligent beings, and produce their effects in a manner fimi-

 lar

lar to the operation of the laws of nature or properties of matter, in cases where thought is not supposed to be concerned, are called final causes.
In the works of nature we behold enough of ex- B
quisite contrivance, and can see far enough into many final causes to convince us that the arrangement of the universe has been made, and probably is still occasionally adjusted, by a being whose intelligence and power is immensely beyond that
which we possess. To judge properly of his in- C
tentions, or in other words, to be equal to the task of exploring the science of final causes, requires no less than a perfect knowledge and recollection of every purpose to which the objects around us may be applied, together with a clear conception of the ideas of fitness and order that form the prototypes in the mind of that great being who directs their
motions. These considerations shew the absurdity D
of attempting to explain the final causes of every event we see, but they by no means require that we should neglect them in cases where we have reason to believe we understand the phenomena, and have sufficient experience to be assured that we discern the principal, or at least one of the principal purposes to which things may have been destined. Thus, it is scarcely to be imagined that we can err in concluding, that the eyes, ears, legs, wings, and other parts of animals were made for the purposes of seeing, hearing, walking, flying, and the like. Neither can we avoid inferring, that the power who constructed living creatures with

mouths, teeth, and organs to digeſt and aſſimilate food for their nutriment, did likewiſe form other organized bodies, which we call vegetables, for the expreſs purpoſe of affording that food. It is needleſs to multiply inſtances. We cannot avoid ſeeing them every moment, and their effect is ſo ſtriking, that we are inſenſibly forced from analogy to allow the exiſtence of a final cauſe in all caſes, whether we are able to diſcover it or no.

On this ground, an enquiry into the final cauſes of the planetary bodies offers itſelf to our conſideration. The earth is ſhewn to be a planet in circumſtances very ſimilar to the other five: we know its final cauſe—to ſupport a number of inhabitants. And by analogy, we may conclude, that the others are alſo habitable worlds; though from their different proportions of heat it is credible, that beings of our make and temperature could not live upon them. However, even that can ſcarcely be affirmed of all the planets: for the warmeſt climate on the planet Mars is not colder than many parts of Norway or Lapland are in the ſpring or autumn. Jupiter, Saturn, and the Georgium Sidus, it muſt be granted, are colder than any of the inhabited parts of our globe. The greateſt heat on the planet Venus, exceeds the heat in the iſland of St. Thomas on the coaſt of Guinea, or Sumatra, in the Eaſt Indies, about as much as the heat in thoſe places exceeds that of the Orkney iſlands, or that of the city of Stockholm in Sweden: therefore,

fore, at 60 degrees north latitude on that planet, if its axis were perpendicular to the plane of its orbit, the heat would not exceed the greatest heat of the Earth, and of course, vegetation like ours might be there carried on, and animals of the species on Earth might subsist. If Mercury's axis be supposed to have a like position, a circle round each pole of about 20 degrees diameter would enjoy the same temperature as the warmer regions of the Earth, though in its hottest climate water would continually boil, and most volatile compounds would be parched up, destroyed or dissipated into vapor. But it is not at all necessary that the planets should be peopled with animals like those on the Earth: the Creator has doubtless adapted the inhabitants of each to their situation.

From the observations that have just been made, a better notion may be formed of the proportions of heat on the planets than can be conveyed by numbers. It will not however be remote from our purpose to compare the light of the superior planets with that of our day, from whence it will appear, that they are by no means in a state of darkness, notwithstanding their great distance from the Sun. This might be instanced by several different methods, as by the Sun's light admitted into a dark chamber, and received on paper with different degrees of obliquity; by a greater or less number of candles brought into a room for the purpose of illuminating it with different proportions of light; or by various optical methods that need not be

here deſcribed. It will be ſufficient for the illuſtration of the ſubject to compare their different proportions of light with that of a moonſhine night at the time of the full.

G When the Moon is viſible in the day-time, its light is ſo nearly equal to that of the lighter thin clouds, that it is with difficulty diſtinguiſhed amongſt them. Its light continues the ſame during the night; but the abſence of the Sun, ſuffering the aperture or pupil of the eye to dilate itſelf, renders it more conſpicuous. It therefore follows, that if every part of the ſky were equally luminous with the Moon's diſc, the light would be the ſame as if in the day-time, it were covered with the thin clouds above-mentioned. This day-light is conſequently in proportion to that of the Moon, as the whole ſurface of the ſky, or viſible hemiſphere, is to the ſurface of the Moon; that is to ſay, near-
H ly as 90,000 to 1. The light of the Georgium Sidus being to that of the Earth as 0.276 to 100, will be equal to the effect of 248 full Moons. The light of Saturn will be equal to that of 990 full Moons: Jupiter's day will equal the light of 3330 Moons, and that of Mars will require 38,700, a number ſo great, that they would almoſt touch one another. It is even probable, that the Comets, in the moſt diſtant parts of their orbits, enjoy a degree of light much exceeding moonſhine.

I If the Comets be habitable they muſt be poſſeſſed by creatures very different from any we have been uſed to behold and conſider. There may, however,

however, be other uſes for which it is probable they may have been formed: the matter that compoſes their tails muſt fall in proceſs of time to the Sun or the neareſt planet that may paſs through it, where it may ſupply defects, and anſwer purpoſes which our total ignorance of its properties ſcarcely allows us even to conjecture. In the Sun it may ſerve to recruit the waſte of matter that luminary may ſuffer by the conſtant emiſſion of the particles of light. After a great number of revolutions, the reſiſtance of the Sun's atmoſphere, and a concurrence of circumſtances, may occaſion the comet itſelf to approach the Sun, and at length fall into it, and become a part of its body.

CHAP. XI.

OF THE TELESCOPIC APPEARANCE OF THE MOON.

THE obſervations which might confirm the hypotheſis of planetary worlds ſeem to be placed beyond our power. We can ſcarcely hope to make optical inſtruments ſufficiently perfect to render their inhabitants viſible to us. The groſs air that ſurrounds us, is a great impediment to the uſe of thoſe we already poſſeſs, and limits their perfection to a certain degree, beyond which we cannot paſs. All, therefore, that we can do, is to examine if the planets are accommodated with thoſe things which we are uſed to conſider as

 neceſſary

neceſſary to animal exiſtence. Lands, ſeas, clouds, vapours, and an atmoſphere or body of air, are objects that we may expect to find on the face of a habitable world: it is our preſent buſineſs to relate what has been done in this reſpect.

L The Moon being ſo very near us, and likewiſe in the ſame temperature as to light and heat, offers itſelf as the fitteſt body for examination. We diſcern a variety of ſpots with the naked eye, which the imagination naturally ſuppoſes to be ſeas, continents, and the like; but on a more accurate inſpection, with the aſſiſtance of the teleſcope, it is perceived that many of thoſe appearances are occaſioned by vaſt obſcure pits or cavities, and elevations or mountains. The heights of theſe mountains may eaſily be found; for by the horizontal parallax we know that the Earth's apparent diameter, ſeen from the Moon at its mean diſtance, is $1^{\circ}\ 54'$ (132, x) or 6840 ſeconds, while that of the Moon ſeen from the Earth at the ſame diſtance

M is $31'\ 29''$, or 1889 ſeconds. Their abſolute diameters muſt therefore be in proportion to theſe numbers. Conſequently, if we find the proportion the height of a lunar mountain bears to the Moon's diameter, we may, without difficulty, find the quantity of that height in miles or other terreſtrial dimenſions.

N Theſe mountains and cavities are known to be ſuch from their ſhadows. In the firſt and ſecond quarters, when the Sun ſhines obliquely on the face of

of the Moon, the elevated parts caſt a triangular ſhadow in the direction from the Sun; and, on the contrary, the cavities are dark on the ſide next the Sun, and illuminated on the oppoſite ſide. The ſhadows ſhorten as the Sun becomes more directly oppoſed to the anterior face of the Moon, and at length diſappear at the time of the full. During the third and laſt quarters, the ſhadows appear again, but all fall towards the contrary ſide of the Moon, though ſtill with the ſame diſtinction, namely, that the mountains are dark and ſhady on the ſide furtheſt from the Sun, and the pits are dark on the ſide next the Sun. The ſame deduction is obtained by contemplating the inner illuminated edge of the Moon. If the Moon were a perfect ſphere, this edge would be a regular curve, but if its ſurface be diverſified by hills and cavities, it is evident that the higher parts muſt be enlightened ſooner, and the cavities later than the reſt of the ſurface. This is accordingly the caſe, and affords a method of obtaining the heights of the mouutains.

To render the explanation eaſier, we ſhall ſuppoſe the Moon to be in its quadrature, and the mountain to be ſituated near the Moon's apparent center.

Let the circle A B D G (fig. 61) repreſent the Moon, whoſe center is C; and E the Earth: then a ſpectator at E will ſee the Moon enlightened in the half A G D, and the line E C will paſs through A, or the inner enlightened edge. The ray of light S A B touching the Moon at A, will croſs the line E C at right

right angles, and illuminate the top of the mountain B. The angle A E B is found by obſervation, then in the triangle A E B,

As the co-ſine of the obſerved angle - A E B
Is to the Moon's diſtance - - A E,
So is the ſine of the obſerved angle - A E B,
To the ſide or line - - - A B.

Then in the right angled triangle C A B, the ſides C A and A B being known, the ſide C B is found from the well known property (Euclid I. 47); that is to ſay, the ſquare of the Moon's ſemidiameter C A being added to the ſquare of the line A B, the ſquare root of the ſum is the ſide C B. And if the ſemidiameter of the Moon C F be taken from the line C B, the remainder is F B, or the height of the mountain.

P From obſervations of this kind,* it appears that the lunar mountains are much higher in proportion to its radius than any we have upon the Earth. Herſchel has obſerved ſeveral volcanos in theſe mountains.

Q That the Moon is ſurrounded by an atmoſphere or body of air, is rendered probable by many obſervations of ſolar eclipſes, in which the limb or edge of the Sun was obſerved to tremble juſt before the beginning. The planets likewiſe are obſerved to change their figure from round to oval juſt before the beginning of an occultation behind the Moon; which can be attributed to no other cauſe, than that their light is refracted, being ſeen through the Moon's atmoſphere. Many aſtronomers are of opinion, that the Moon has no atmoſphere,

ſphere, becauſe we ſee no clouds, and becauſe the fixed ſtars diſappear at once at the time of an occultation without any gradual diminution of light, which they ſuppoſe ought to take place. But if we conſider the effect of days and nights near thirty times as long as with us, we may readily grant that the phenomena of vapours and meteors may be very different: perhaps their clouds and rain, if any, may be condenſed into viſible quantities, only during the abſence of the Sun, and if ſo, it is no wonder that we never ſee them. With reſpect to the fixed ſtars, it is plain, that granting the Moon to have an atmoſphere of the ſame nature and quantity as ours, no ſuch effect as a gradual diminution of light ought to take place, at leaſt as to ſenſe. Our atmoſphere is found to be ſo rare at the height of 44 miles, as to be incapable of acting on the rays of light. This height is the 180th part of the Earth's diameter; but ſince clouds are never obſerved higher than four miles, we muſt conclude, that the vaporous or obſcure part is but the 1980th part. The mean apparent diameter of the Moon is 31′ 29″, or 1889 ſeconds; therefore, the obſcure part of its atmoſphere, when viewed from the Earth, muſt ſubtend an angle of leſs than one ſecond, which ſpace is paſſed over by the Moon in leſs than two ſeconds of time; a ſpace and time ſo ſhort, that it can hardly be expected that obſervation can in general determine whether the ſuppoſed obſcuration takes place or not.

It

R It must, however, be allowed, that the Moon's atmoſphere, if it has one, is certainly much leſs conſiderable than that of the Earth. For there is ſeldom any appearance, in an eclipſe of the Sun, of the remarkable phenomenon (158, Q), that the Earth's atmoſphere is ſhewn to produce in a lunar eclipſe*.

S The Moon turns round on its own axis once in the time of its periodical revolution. This is evident, becauſe the ſame face or ſide is conſtantly turned towards us. For a ſpectator on the Moon will ſee the Earth carried through every part of the ecliptic in the courſe of one revolution; and as the ſame face of the Moon is conſtantly turned towards the Earth, it muſt be ſucceſſively turned to every part of the ecliptic to which the Earth apparently moves. But if it be ſucceſſively turned to every part of a great circle in the heavens, it muſt revolve on its axis. By this ſlow rotation, it appears, that the inhabitants of the Moon have but one day and night in the courſe of a month.

T This rotation on its axis is the moſt uniform motion the Moon has; but its uniformity occaſions a ſeeming irregularity, which is termed the libration. For as the Moon's motion in its orbit was ſhewn to be not uniform (143, F) the effect it has in turning its face from the Earth is likewiſe ſubject to the ſame irregularities; for inſtance, in the ſwifteſt part of the revolution, its motion in its orbit turns its face

* See Philoſophical Tranſactions, Anno, 1779, for an account of the ſolar eclipſe of June 24, 1778, in which this appearance was ſeen by Don Antonio Ulloa.

from

from the Earth ſomething more than the rotation on its axis turns it the other way, and therefore it appears to have a ſmall motion on its axis towards the eaſt, by which ſome of the more weſtern parts are brought to view, and an equal portion of the eaſtern limb diſappears. In the ſlower part the contrary is ſeen, for then the rotation on its axis prevailing, brings the weſtern parts into view, and the eaſtern diſappear. This is called libration in longitude.

There is another kind of libration that ariſes from the Moon's axis being inclined to the plane of its orbit, by which means ſometimes one of its poles, and ſometimes the other, is inclined towards the Earth. In conſequence of this, we ſee more or leſs of the polar regions at different times. This is called libration in latitude.

CHAP.

CHAP. XII.

OF THE TELESCOPIC APPEARANCES OF THE SUN AND THE PLANETS.

The Sun is not without ſpots on its diſc, but they are ſeldom ſo large as to be ſeen by the naked eye. When viewed with a teleſcope, the eye being defended by a piece of coloured or ſmoked glaſs, they are found to appear in various forms and numbers. The larger ſpots, moſt of which exceed the whole Earth in apparent magnitude, laſt a conſiderable time; ſometimes three months before they diſappear, at which time they are generally converted into faculæ, or ſpots which exceed the reſt of the Sun in brightneſs. They are of no conſtant figure, frequently changing during the time of obſervation, and ſometimes one dividing into ſeveral ſmaller ones. In general they conſiſt of a nucleus or central part, much darker than the reſt, which is ſurrounded by a miſtineſs or ſmoke. The general opinion concerning them is, that they are occaſioned by the ſmoke and opake matter thrown out by volcanos or burning mountains of immenſe magnitude; and that when the eruption is nearly ended, and the ſmoke diſſipated, the fierce flames are expoſed and appear as faculæ, or luminous ſpots. But Dr. Alexander Wilſon of Glaſgow has eſtabliſhed from obſervation, that moſt, if not all the ſpots, are excavations in the luminous matter

that

that environs the body of the Sun, probably to no great depth*. At present (anno 1779) they are often seen to the number of thirty or more, but there have been periods of more than seven years, in which none have been observed.

All the spots of the Sun have an apparent motion w from east to west, which is quicker when they are near the central regions than when near the limb. This proves that the sun revolves on its axis from x west to east, and likewise that its figure is spherical. The period, as observed by Cassini, is 25 days, 14 y hours, 8 minutes. From the line of the motion of the spots, which is sometimes strait, but oftener curved or elliptical, it is discovered that its axis is z not perpendicular to the ecliptic, but inclined, so as to make an angle with the perpendicular of about seven degrees and a half.

The zodiacal light is attributed to the solar atmo- A sphere. This remarkable phenomenon accompanies the Sun. When it begins to appear before sun-rise, it seems at first sight like a faint, and almost imperceptible, whitish light, resembling the milky-way, and ill-terminated, which is almost confounded with the twilight that is seen commencing near the horizon. It is then little elevated, and its termination may sometimes be discerned in a conic or conoidal form. In short, its figure agrees with that of a very flat or lenticular spheroid seen in profile. As it gradually rises above the horizon it becomes brighter and larger to a certain point, that may be call-

* Philosophical Transactions, 1774.

ed

ed its maximum, after which the approach of day renders it gradually lefs apparent, and at laft invifible. The direction of its longer apparent axis is obferved to be in the plane of the Sun's equator; but its length is fubject to great variation, fo that the diftance of its fummit from the Sun varies from 45° to even 120°. Thefe great differences in magnitude and brightnefs may perhaps depend confiderably on fome, yet unknown, circumftances in our atmofphere. It is ufually feen to the greateft advantage about the folftice.

B It has been fuppofed, that this atmofphere is the caufe of the afcent of the vapor which forms the tails of the comets, and which is always carried to that part of the heavens which is oppofite the Sun. But the direction of thefe vapors may perhaps be determined by the action of the particles of light emitted from the Sun.

C The planet Mercury is at all times fo near the Sun, that we can only diftinguifh with the telefcope a variation in its figure, which is fometimes that of a half Moon, and fometimes a little more or lefs than half. Whence it follows, that its form is globular, and that it receives all its light from the Sun.

D The planet Venus, when viewed through the telefcope, has a very pleafing appearance. At the time of its greateft elongation it appears like the Moon in the quadratures; one half of its difc being enlightened. In the inferior part of its orbit, as its elongation decreafes, the enlightened part becomes lefs,

leſs, appearing falcated or horned; after paſſing the inferior conjunction, the planet is again ſeen horned, but the illuminated part then increaſes, and at the greateſt elongation, half its diſk is again ſeen enlightened. In the ſuperior part of its orbit, as its elongation decreaſes, its face becomes more full and round, till the ſuperior conjunction, after which time it is again diminiſhed by the ſame gradation as its increaſe was in the former caſe accompliſhed. There is no difficulty in accounting for this variety of phaſes, it being occaſioned by the different poſitions of Venus with reſpect to the Sun and Earth: for as the enlightened face of Venus muſt of courſe be always oppoſite to or facing the Sun, it will be more or leſs viſible to us according to our ſituation at various times.

The ſurface of Venus is diverſified with ſpots like E
our Moon, by the motion of which it is determined, that it revolves on its axis from weſt to eaſt in the ſpace of twenty-three hours. When the air is in a good ſtate for this kind of obſervations, mountains like thoſe in the Moon may be diſcerned, with a very powerful teleſcope. Late obſervations of Herſchel have however rendered theſe accounts uncertain *.

The face of the planet Mars is always round and F
full, as its ſuperior ſituation requires, excepting at the time of the quadrature, or elongation of 90 degrees, when a ſmall part of the unenlightened hemi-

* Philoſophical Tranſactions. 1793.

ſphere being turned towards us, its diſc appears like the Moon about three days after the full.

G By the ſpots on Mars, its diurnal revolution is aſcertained in the direction from weſt to eaſt. From the ruddy and obſcure appearance of this planet, as well as from other appearances, it is concluded, that its atmoſphere is nearly of the ſame denſity as that of the Earth. Mr. Herſchel has obſerved, that two circles ſurrounding the poles of this planet are very white, and luminous, probably, from ſnow lying there.

H We have already had occaſion to ſpeak of the ſatellites of Jupiter and Saturn. The annual parallax of theſe planets is not conſiderable enough to bring any ſenſible part of their dark hemiſpheres towards us in any poſition of elongation; conſequently their faces are always round and full.

I The teleſcopic appearance of Jupiter affords a vaſt field for the curious enquirer. It is in general encircled with one or more obſcure belts or bands parallel to the plane of its orbit, and conſequently to each other. Theſe are not regular or conſtant in their appearance. They have been ſeen to the number of five, and during the time of obſervation two have gradually diſappeared. Sometimes but one is ſeen; and ſometimes, when the number is more conſiderable, one or more dark ſpots are formed between the belts, which increaſe till the whole is united in one large duſky band. The ſpots of Jupiter are the brighter parts of its ſurface, and are not permanent, though more ſo

 than

than the belts; yet it is found that they re-appear after certain unequal intervals of time. The remarkable ſpot, by whoſe motion the rotation of Jupiter on its axis was determined, diſappeared in 1694, and was not ſeen again till 1708, when it re-appeared exactly in the ſame place on its ſurface, and has been occaſionally ſeen ever ſince.

It has been conjectured, that theſe belts are K ſeas, and that the variations obſerved both in them and the ſpots are occaſioned by tides, which are differently affected, according to the poſitions of his moons. It is probable, however, that they are in its atmoſphere. If an obſerver, poſſeſſed of ſkill and patience equal to the taſk, would delineate the phaſes of Jupiter for the ſpace of a periodical revolution, noting at the ſame time the poſitions of his ſatellites, this opinion might be either eſtabliſhed or rejected: but at all events ſuch a ſeries of obſervations could not fail to throw great light on the ſubject.

The very great diſtance of the planet Saturn, and L the tenuity of its light, do not permit common obſervers to diſtinguiſh thoſe varieties which it is probable are on its ſurface. Herſchel's teleſcopes ſhew belts on its ſurface. Theſe are generally parallel to the ring. Saturn is found to revolve on an axis perpendicular to the plane of the ring, and its diameter is ſhorter than that which is meaſured in the plane of the ring in the proportion of 10 to 11. The ring itſelf is inclined to the ecliptic; in conſequence of which, its ap-

parent figure is continually varying. When the line of its nodes points directly towards the Earth, the ring, preſenting its edge to the obſerver, becomes inviſible to common teleſcopes: if the ſame line points directly towards the Sun, the ring becomes inviſible for want of illumination: and laſtly, if the plane of the ring paſſes between the Sun and the Earth, the ring cannot then be ſeen, becauſe its dark ſide is towards us. At all other times its figure is that of an oval, which is broader or narrower accordingly as the line of the nodes is farther from or nearer to the above poſitions.

CHAP. XIII.

OF THE LENGTH OF DAYS AND NIGHTS; AND OF THE SEASONS.

M WE have ſeen that every planet which is acceſſible to obſervation has a revolution on its axis; the intention of which is, undoubtedly, to give alternate night and day to every part of their ſurfaces. An inclination of the axis of any planet to the axis of its orbit, by cauſing the length of days and the intenſity of heat to vary, will occaſion a viciſſitude of ſeaſons. On this account Jupiter, whoſe axis is nearly perpendicular to the plane of its orbit, has equal days and nights on every part of its ſurface at the ſame time, the days being four hours and twenty-eight minutes, and the nights of the ſame length. But the planets Mars and Venus, whoſe axes are

inclined

inclined to the planes of their reſpective orbits, have each an annual change of ſeaſons and length of days. The Earth, for the ſame reaſon, has a ſimilar viciſſitude, the explanation of which will render it unneceſſary to enlarge on the circumſtances of the other planets.

For this purpoſe it will be uſeful to define thoſe N
imaginary circles, which aſtronomers and geographers have invented for the purpoſes of methodizing and facilitating the communication of ſcience.

On the Earth, a great circle, ſuppoſed to be O
drawn at an equal diſtance from each pole, is termed the Equator: leſs circles drawn parallel to the equator are called Parallels of Latitude; and great circles interſecting the equator at right angles, and conſequently paſſing through the poles, are called Meridians. But when the meridian of a place is ſpoken of, it is uſually underſtood to be a ſemicircle paſſing through the given place, and terminating at the poles. The other half which completes that whole circle, is then called the oppoſite meridian.

In the heavens, a great circle, parallel to the P
equator, is termed the Celeſtial Equator; but the leſs circles parallel to it are called Parallels of Declination; and the great circles interſecting it at right angles, and paſſing through the celeſtial poles of the Earth, are called Hour Circles, or circles of right aſcenſion.

The ecliptic is that great circle in the heavens, Q
in which the Sun deſcribes its apparent annual

courſe: leſs circles, drawn parallel to the ecliptic, are called Parallels of Latitude; and great circles interſecting it at right angles, and conſequently paſſing through its poles, are called Celeſtial Meridians.

R The horizon is that great circle which divides the viſible or upper hemiſphere from the lower. If this circle have the eye of the obſerver for its center, it is called the Senſible Horizon; but if its center be that of the Earth, it is termed the Rational Horizon. To this laſt all aſtronomical obſervations are reduced or applied; the former being only conſidered as one of the parallels of altitude. Leſs circles, parallel to the horizon, are called Parallels of Altitude, if above, but of Depreſſion, if below the horizon, and the great circles interſecting it at right angles, are called Azimuths.

S The point of the heavens, which is immediately above the obſerver, or is elevated 90° above the horizon, is termed the Zenith; the oppoſite point immediately beneath, or at 90° of depreſſion below the horizon, is termed the Nadir.

T Latitude on the Earth is an arc of the meridian, contained between a given place and the equator. It is meaſured in degrees and minutes of the meridian. In the heavens it is an arc of the celeſtial meridian, contained between a given place and the ecliptic.

U Longitude on the Earth is an arc of the equator, contained between the meridian paſſing through a given place and the firſt meridian. It never exceeds a ſemicircle. The firſt meridian on the Earth

is

is arbitrary; but the Engliſh aſtronomers in general reckon from that which paſſes through the obſervatory at Greenwich. Longitude in the heavens is an arc of the ecliptic, contained between a given meridian and that which paſſes through the firſt point of the conſtellation Aries; the ſaid point being always at the weſtern extremity of the arc.

Right aſcenſion is an arc of the celeſtial equator, v
contained between a given hour circle and that which paſſes through the firſt point of the conſtellation Aries; the ſaid point being always at the weſtern extremity of the arc. Declination is an arc of the hour circle, contained between a given place and the equator.

The circle which divides the enlightened hemi- w
ſphere of a planet from its dark hemiſphere is called the Terminator. It may in moſt caſes be conſidered as a great circle.

Let N E S Q (fig. 62 and 63) repreſent the Globe x
of the Earth, and C the Sun: then the circles N M S, N M S, &c. will repreſent the meridians interſecting the Equator E Q at right angles, and paſſing through the poles N and S. The lines pp, pp, &c. will repreſent the parallels of latitude; and the line C M will repreſent the plane of the Earth's orbit.

Now it is evident, that it is day at any given Y
place on the globe, ſo long as that place continues in the enlightened hemiſphere; and that when by the diurnal rotation it is carried into the dark hemiſphere it becomes night; twilight not being here conſidered. And from the contemplation of figure

63, it appears, that if the poles be ſituated in the terminator, the terminator will divide each of the parallels into two equal parts, and conſequently, ſince the uniform motion of the Earth cauſes any given place to deſcribe equal parts of its parallel in equal times, the days and nights will be equal on every parallel of latitude; that is to ſay, all over the globe, except at the poles, where the Sun will neither riſe nor ſet, but continue in the horizon.

But if, as in figure 62, the axis be not placed in the plane of the terminator, the terminator will divide the equator into two equal parts; but the parallels which are ſituated towards the enlightened pole will have a greater part of their peripheries in the enlightened than in the dark hemiſphere: while ſimilar parallels towards the other pole will have a like greater part of their peripheries in the dark hemiſphere. Whence it follows, that the firſt-mentioned parallels will enjoy longer days than nights, and the contrary will happen to the latter, they having ſhorter days and longer nights; while at the equator the days and nights continue equal. All this is evident from the figure, where it is alſo obſervable, that the diſproportion is greateſt in the greater latitudes; and that places, whoſe diſtance from the pole is leſs than that of the pole from the terminator, muſt enjoy either a conſtant day or conſtant night, the rotation of the Earth never carrying them into the oppoſite hemiſphere.

In this poſition of the axis the inhabitants on the one ſide of the equator may be ſaid to enjoy ſum-

mer, and thoſe on the other ſide winter with reſpect to each other. For the long duration of the Sun above the horizon muſt occaſion a proportionally greater degree of heat, and its longer abſence from places ſituated in the other hemiſphere muſt have the contrary effect.

But this is not the only cauſe of the difference of B
heat at the different places. A ſpectator at G, which is 90° diſtant from the terminator, will have the Sun in the zenith; a ſpectator at T will ſee the Sun in the horizon; and, for every intermediate diſtance, the arc of a great circle comprehended between the terminator and the place of obſervation will be the meaſure of the Sun's altitude. Therefore every parallel between Q and the enlightened pole will have the meridian altitude of the Sun increaſed, by the angle N M T, beyond what it would have been had the pole continued in the plane of the terminator: and every place between Q and the dark pole will have the Sun's meridian altitude diminiſhed by the ſame quantity. Between G and Q the former altitude thus increaſed, making a ſum greater than 90°, the altitude muſt be meaſured by its complement. And between S and T the altitude will be a negative quantity, or beneath the horizon. This difference of the altitudes of the Sun muſt cauſe an increaſe of heat towards the enlightened pole, and an equal diminution towards the dark pole. For the greater the Sun's altitude, the more directly its rays fall on any ſurface; and C
in ſurfaces of the ſame magnitude the quantity of light

light received by each is as the ſine of the angle of obliquity with which the rays fall*.

D It remains to be ſhewn, that theſe relative ſituations of the axis and the terminator take place at different times of the year, with reſpect to the Earth; which being proved, the viciſſitude of ſeaſons muſt follow as a neceſſary conſequence.

E In fig. 65. Let C repreſent the Sun, A B D G the Earth's orbit, nearly circular, but which being viewed obliquely, appears like a long ellipſis, of which let the part B D be ſuppoſed neareſt the ſpectator. And let the four circles, diſtinguiſhed by the months March, June, September, and December, repreſent the Earth in four ſeveral parts of its orbit, N S being its axis.

F Obſervation ſhews, that the axis of the Earth always preſerves very nearly the ſame poſition with reſpect to the fixed ſtars; being inclined to the axis of its orbit in an angle of about $23\frac{1}{2}$ degrees. It will not therefore preſerve the ſame relative poſition with reſpect to the terminator. For when the Earth is in the ſituation diſtinguiſhed by the

* Let the line A B (fig. 64) repreſent a ſurface, on which the column of light N O A B falls perpendicularly. A ſurface A C, of the ſame magnitude, receiving the light obliquely under the angle J C K, will intercept only ſo much as would have fallen on the ſpace A E and another ſurface A D, receiving the light ſtill more obliquely under the angle L D M, will intercept only ſo much as would have fallen on the ſpace A F. But the ſpaces or lines A E and A F are the ſines of the angles of obliquity J C K and L D M; whence the propoſition is evident.

month

month March, its axis will at that time be in the plane of the terminator, and conſequently the days and nights will be equal all over the globe (183, Y): but when by its annual motion it is carried towards A, the north pole N, the axis ſtill preſerving its poſition or continuing parallel to itſelf, will advance into the enlightened hemiſphere, and in the month of June will be $23\frac{1}{2}$ degrees diſtant from the terminator, as in the ſcheme, the ſouth pole being at the ſame diſtance in the dark hemiſphere. Therefore in the month of June the northern parts will enjoy long days and ſummer, while the ſouthern parts have ſhort days and winter (184, Z).

During the interval between the time of equal days and nights in March, which is called the vernal equinox, and the time when the day is longeſt in June, which is called the ſummer ſolſtice, the north pole will have deſcribed a quarter of a circle in the enlightened hemiſphere with reſpect to the terminator, and conſequently will be at its greateſt diſtance from it. From that time it will, by deſcribing the other quarter, approach the terminator, the days gradually ſhortening till the Earth arrives at the poſition denoted by the month September, when, the axis again coinciding with the plane of the terminator, the days and nights will be equal. This is called the autumnal equinox. During the next quarter the north pole will deſcribe a quarter of a circle in the dark hemiſphere, and the days will ſhorten till December, when the pole will be juſt as far within the dark

as in June it was in the enlightened hemiſphere; which time is called the winter ſolſtice. From the winter ſolſtice to the vernal equinox, the days will lengthen as the pole approaches the terminator; and at the inſtant in which the axis again coincides with its plane, the natural year, conſiſting of 365 days, 5 hours, 48 minutes, and $45\frac{1}{2}$ ſeconds, is finiſhed.

H It is eaſy to conceive, by applying the ſame explanation to the ſouth pole inſtead of the north, that the inhabitants of the ſouthern hemiſphere have the ſame viciſſitudes, though not at the ſame time; for it is winter in one hemiſphere while it is ſummer in the other, &c. &c.

I As the pole N (fig. 62) advances in the enlightened hemiſphere, the Sun will be in the zenith of a place G, as far diſtant from the equator, as the pole is from the terminator; therefore the greateſt latitude at which the Sun can be vertical is $23\frac{1}{2}$ degrees. The parallels of latitude on the Earth of $23\frac{1}{2}$ degrees N. and S. as alſo the correſpondent parallels of declination in the heavens, are called the Tropics, becauſe the Sun when it arrives at them afterwards returns towards the equator. The Sun, when it arrives at the northern tropic, is juſt entering the ſign Cancer, and when it arrives at the ſouthern tropic is juſt entering the ſign Capricorn; for which reaſon the northern tropic is called the Tropic of Cancer, and the ſouthern tropic the Tropic of Capricorn.

CHAP.

CHAP. XIV.

OF THE FIXED STARS.

THOUGH in a former chapter of this ſection it K was mentioned (104) that the relative ſituations of the fixed ſtars do not vary, yet that aſſertion is not to be underſtood in abſolute ſtrictneſs. In the courſe of ages ſeveral variations have been obſerved amongſt them. Some of the larger ſtars have not the ſame preciſe ſituations that ancient obſervations attribute to them; and it is probable that the inſtances of this kind would have been much more numerous if accuracy of obſervation had not been confined to a very late period. New ſtars have likewiſe appeared from time to time, and ſeveral of thoſe whoſe places and magnitudes are inſerted in the old catalogues are not now to be found. Some of the fixed ſtars are likewiſe found to have a periodical increaſe and decreaſe of magnitude.

The bright ſtars Arcturus, Sirius, Aquila, and L Aldebaran, have been obſerved to change their places. The firſt is found to move towards the ſouth, about $3\frac{1}{4}$ minutes of a degree in a century. Sirius has advanced about 2 minutes to the ſouth in a like period. The changes of place in the two latter are yet ſmaller and leſs ſettled.

All the ſtars ſpoken of in the preſent chapter M are ſubject to no parallax (130, z), according to the

the moſt accurate obſervations by which moſt of their places were ſettled; and ſome of them have been obſerved with inſtruments of ſuch delicacy, that it is preſumed their parallax would have been ſeen, if it had amounted to one ſecond of meaſure.

N Without attending either to the celeſtial changes recorded by ancient authors, who for the moſt part were not aſtronomers, and paſſing the leſs obvious mutations in ſilence, we ſhall here note a few of the moſt remarkable new or changeable ſtars.

O On the 8th of November, 1572, Cornelius Gemma attentively conſidered that part of the heavens which is called Caſſiope's Chair, and perceived nothing extraordinary. But the following night a new ſtar appeared, forming a perfect rhombus with the three ſtars α, β, γ, of that conſtellation. Its ſplendor exceeded that of Jupiter when greateſt, and was ſuch, that it was ſeen even in the day-time. Tycho Brahe, who ſaw it on the 11th, determined its longitude 6° 54′ of Taurus, with 53° 45′ N. latitude. It began to diminiſh in December, and became gradually leſs conſpicuous, till it diſappeared in March, 1574. This remarkable ſtar had no apparent motion, and conſequently no parallax, and its appearance was ſparkling and clear, like that of the other fixed ſtars. It has not ſince been ſeen.

P September, 1604, O. S. the ſcholars of Kepler obſerved a ſtar in the right leg of Serpentarius, which

was

was not there the night before. Its lustre seems to have been nearly equal to that of the new star in Cassiope; for it is described as exceeding Jupiter in brightness. It gradually decayed like that, and in nearly the same time disappeared, not being perceived after the beginning of January, $160\frac{4}{5}$. Its right ascension, as observed by Kepler, was constantly 256° 57′, and its declination $2101\frac{1}{2}$ S.

The first star that was observed to have a periodical change of brightness was discovered by David Fabricius in the neck of the Whale, on the 3d of August, 1596, O. S. Its greatest brightness is equal to that of a star of the third magnitude; and it is scarcely ever so small but it may be seen with a six foot telescope. The period in which it passes through all its changes, is at a mean 334 days, but no part of the phenomenon is perfectly regular. Q

Three changeable stars have been observed in the constellation of the Swan. The first discovered is near the star γ in that constellation. Its greatest lustre is less than that of a star of the third magnitude, and it diminishes to that of the sixth magnitude. Its changes are far from being regular, and do not take place but after intervals of ten or more years. R

The next and most remarkable of the changeable stars in the Swan is marked χ by Bayer. This is more regular in its returns than the preceding, through its magnitude is seldom greater than the sixth. Its period is settled at $405\frac{1}{16}$ days, and its greatest lustre in the year 1785 was about the 14th of July. S

The

T The third was ſeen near the head of the Swan on the 20th of June, 1670, of about the third magnitude, and was ſo far diminiſhed by the October following as to be ſcarcely viſible. In the beginning of April, 1671, it was again ſeen rather brighter than before, and diminiſhing during that month, became once more at its greateſt brilliancy at the beginning of May. By a compariſon of theſe obſervations, its period ſeemed about ten months. It diſappeared about the middle of Auguſt, and was again ſeen on the 29th of March, 1672; ſince which time it has not appeared.

U The ſtar Algol, or Meduſa's head, has been long ſince obſerved to appear of different magnitudes at different times; but the diſcovery of its period is due to John Goodricke, Eſq; of York, who has obſerved it ſince the beginning of the year 1783. It periodically changes from the firſt to the fourth magnitude; and the time employed from one greateſt diminution to the other, was Anno 1783, at a mean, 2 days, 20 hours, 49 minutes, 3 ſeconds. The change is thus. During four hours it gradually diminiſhes in luſtre; during the ſucceeding four hours it recovers its firſt magnitude by a like gradual increaſe; and during the remaining part of the period, namely, 2 days, 12 hours, 42 minutes, 3 ſeconds, it invariably preſerves its greateſt luſtre: after the expiration of which term the diminution again commences, &c.

V Many of the fixed ſtars, upon examination with the teleſcope, are found to conſiſt of two. The number

number obſerved before the time of Herſchel* was but ſmall; but that celebrated aſtronomer, who ſtands unrivalled for the excellence of his inſtruments, and his ſkilful induſtry in uſing them, has noted upwards of four hundred.

Beſides the phenomena already mentioned, there w
are many nebulæ, or parts of the heavens which are brighter than the reſt. The moſt obvious to common notice is that large irregular zone or band of light which croſſes the ecliptic in Cancer and Capricorn, and is inclined to it in an angle of about ſixty degrees. Other nebulæ are ſeldom ſo large as to be ſeen by the naked eye, to which they appear as ſmall ſtars. If the teleſcope be applied to them, they ſeem to be luminous ſpots of various figures, in ſome inſtances with ſtars in them. The number of nebulæ aſcertained before Herſchel are about 103, and that obſerver had detected 466 more, previous to the month of April 1784. Many of the nebulæ are reſolvable by the teleſcope into cluſters of ſmall ſtars; and it is found that teleſcopes of greater power reſolve thoſe nebulæ into ſtars, which appear as white clouds in inſtruments of leſs force. Hence there is good reaſon to conclude that they all conſiſt of cluſters or prodigious aggregates of ſtars.

Dr. Herſchel has rendered it highly probable, x
both from obſervation and well-grounded conjecture, that the ſtarry heaven is replete with theſe nebulæ or

* Dr. Herſchel's numerous and important diſcoveries are inſerted in the late volumes of the Philoſophical Tranſactions, as are alſo the accounts of Algol.

ſyſtems of ſtars of various figures, and that the milky-way is that particular nebula in which our Sun is placed. Nothing more is neceſſary in order to account for the appearance it exhibits, than to aſſume its figure as being much more extended towards the apparent zone of illumination than in other directions. And the obſervations on the various figures of the nebulæ render this ſuppoſition perfectly allowable.

Y The want of an annual parallax in the fixed ſtars evinces, that a luminous body, whoſe diameter is equal to that of the Earth's annual orbit, would not ſubtend a ſenſible angle if ſeen from the fixed ſtars. Much leſs therefore would the Sun, if viewed from ſuch a diſtance. It may therefore be fairly concluded that the Sun, when ſeen from any fixed ſtar, muſt have much the ſame appearance as a fixed ſtar ſeen from hence: or, in other words, the fixed ſtars are ſuns.

Z Reaſoning then analogically, as far as the nature of the facts we poſſeſs will admit, it may be deduced; firſt, that the univerſe conſiſts of nebulæ, or diſtinct ſyſtems of ſtars; ſecondly, that each nebula is compoſed of a prodigious number of ſuns, or bodies that ſhine by their own native ſplendor; thirdly, that each individual ſun is deſtined to give light to hundreds (124, R. 161, W. 164, E) of worlds that revolve about it, but which can no more be ſeen by us, on account of their great diſtance, than the ſolar planets can be ſeen from the fixed ſtars.

Yet,

Yet, as in this unexplored, and, perhaps, unexplorable abyſs of ſpace, it is no neceſſary condition that the planets ſhould be of the ſame magnitudes as thoſe belonging to our ſyſtem, it is not improbable but that planetary bodies may be diſcovered among the double and triple ſtars. A

Our curioſity is much intereſted in the contemplation of the phenomena of new and changeable ſtars, but the cauſes that may be offered with plauſibility to ſolve theſe appearances are not many. C

If the light of the Sun and fixed ſtars be imagined to proceed from a combuſtion ſimilar to that which is required to produce light in our experiments, it may happen that when all the inflammable matter is decompoſed the ignition may ceaſe. Or, if a maſs of matter adapted for inflammation begin by any cauſe to burn, its ignition and emiſſion of light will then commence. Theſe conſiderations may explain the diſappearance of ſome ſtars, and the appearance of others. And as there are no data to fix the time between the beginning and end of the appearance, the ſtars may laſt for any given time, according to circumſtances. B

The ſpots on the Sun (174, v) have afforded a conjecture reſpecting the cauſe of the periodical change of brightneſs in ſome ſtars. For, if a ſtar be ſuppoſed to have a ſpot of conſiderable magnitude, and to turn on its axis, it will be much brighter when the ſpot is not on the viſible diſc than when it is. However, it muſt be confeſſed, that the phenomena do not in general agree with this ſuppoſition, D

which cannot eaſily be reconciled to the permanent brightneſs or obſcurity that prevails in the changeable ſtars for more than half the period.

E If a ſtar, by a ſwift rotation, be made to aſſume and preſerve a flattened figure, and its axis have a rotation ſimilar to that hereafter to be explained in the Earth, it will be much leſs bright when its edge is preſented to the obſerver than when the viſible diſc is projected broader.

F Or, laſtly, if a large planet revolve about a ſtar, it may occaſion certain periodical eclipſes of ſufficient magnitude and duration to be perceived by us, on account of the quantity of light inter-
G cepted. Thus, for example, if an opaque planet, whoſe diameter is not much leſs than that of Algol be ſuppoſed to revolve about that ſtar at the diſtance of thirty-three diameters of Algol, in the given period of 2 days, 20 hours, 49 minutes, 3 ſeconds, in an orbit whoſe plane paſſes at preſent through or near the Earth, it will cauſe certain eclipſes that will agree very well with the appearances obſerved.

BOOK

BOOK I.

SECT. IV.

Concerning the Physical Causes of the Celestial Motions.

CHAP. I.

OF THE GENERAL EFFECTS OF CENTRIPETAL FORCES ON BODIES IN MOTION.

We are now to resume the consideration of bodies in motion, which are acted on by a centripetal force (94, I) and to apply that doctrine to the phenomena explained in the preceding section. A

Let A B C D, &c. (fig. 66) be a regular polygon, inscribed in a circle. If a body be supposed to revolve in that polygon, it will be necessary that a force directed to the center S should be applied at the points B, C, D, &c. to deflect the body from its right lined course (21, P). The more numerous the sides of the polygon are, the nearer they will be to the circular curve, and the more frequent must be the successive actions of the centripetal force. And if the number of sides be infinitely great, the polygon will absolutely coincide with the circle, or become a circle, B

and the actions of the centripetal force must be infinitely numerous, or the force will act without inter-
C mission. Consequently, whatever may be proved in general of a body moving in a regular polygon by an original uniform motion, combined with the motion produced by the successive actions of a force directed to the center of the polygon, will hold good with respect to the motion of a body in a circle, the centripetal force being supposed to act without intermission.

D In our reasoning concerning centripetal forces, it is here supposed that a given force acts on bodies according to their masses, like gravity (26, A) and consequently causes equal deflections in each from the right lined course.

E The intensity or quantity of any force is measured by the effect it produces in a given time (21, Q. 38, T). Suppose a body to be projected from M to A, fig. 66, a centripetal force represented by A R will cause it to describe the line A B instead of A Q in an equal space of time (23, T). If the velocity in M A had been greater, the action of the force in the line A R must have been greater in the same proportion to have caused the same deflection; that is, the centripetal force must at each point of deflection be as the velocity. But again, the greater the velocity the greater number of sides of the polygon will be described by the body in a given time, and the more frequent must be the actions of the centripetal force. For this reason therefore, the number of the actions
F must likewise be as the velocity. On both accounts, therefore,

therefore, the whole effect of the force in a given time, or its intenſity, muſt be in the duplicate proportion, or as the ſquare of the velocity, that is, ſimply as the velocity, becauſe the actions themſelves are greater or leſs in that proportion; and again, ſimply as the velocity, becauſe the actions take place more or leſs frequently in the ſame proportion. And the ſame is true of the G
effect of an unceaſing force that may cauſe the body to revolve in the circle in which the polygon is inſcribed (198, C).

The polygon N O P being ſimilar to the polygon H
A B C, will, with a given velocity, require the ſame action to cauſe the requiſite deflections from the right lined courſe in a body revolving in it. But thoſe actions muſt recur oftener in proportion as the ſide of the polygon N O P is leſs than that of the polygon A B C, becauſe a proportionally greater number of the ſmaller ſides will be paſſed over with the ſame velocity. The force in the ſmaller polygon muſt therefore be increaſed in the inverſe proportion of its ſide to that of the greater, or, which is the ſame, in the inverſe proportion of their radii. On the whole then, the I
centripetal forces, by which bodies are retained in circular orbits, (198, C) are in a ratio compounded of the direct ratio of the ſquares of the velocities, (199, F. G) and the inverſe ratio of their ſemidiameters.

The periodical times of bodies revolving in cir- K
cles are greater, the greater the radii, and leſs, the

greater the velocities. That is to ſay, the periodical times are directly as the radii, and inverſely as the velocities.

L If the centripetal forces be ſuppoſed to increaſe as the cubes or third powers of the radii decreaſe, the cubes of the radii will be inverſely as the ſquares of the velocities, and directly as the radii, becauſe the forces themſelves are inverſely in this compound ratio (199, I). Whence the ſquares of the velocities will be directly as the radii, and inverſely as the cubes of the radii; or more ſimply,
M as the ſquares of the radii inverſely. And the velocities themſelves will conſequently be inverſely as the radii.

N If the ſquares of the periodical times be directly as the cubes of the radii, the cubes of the radii will be (199, K) directly as the ſquares of the radii; and inverſely as the ſquares of the velocities. Therefore, the ſquares of the velocities will be directly as the ſquares of the radii, and inverſely as the cubes of the radii; or, more ſimply, as the radii inverſely. Now the centripetal forces (199, I) are directly as the ſquares of the velocities, and inverſely as the radii. Therefore, if in this laſt compound ratio we ſubſtitute the inverſe ratio of the radii, inſtead of the direct ratio of the ſquares of the velocities, to which it is equal, we ſhall have the forces in the inverſe ratio of the ſquares
O of the radii. That is to ſay, if the ſquares of the periodical times of bodies revolving in circles be

directly as the cubes of the radii, the centripetal forces will be inverſely as the ſquares of the radii.

The velocity of a body moving in a curve, and P
acted on by a centripetal force, is inverſely as the perpendicular let fall from the center, to the tangent drawn through that point of the curve at which the velocity is required. Let A B, fig. 67, be a curve in which a body moves, deſcribing equal areas in equal times (95, M), about the point C. Then the velocity at the points D and F will be inverſely as the perpendiculars C H, C I, let fall from C on the tangents D H, F I to the curve in the points D, F. For if the body move through the ſpaces D E, F G, in equal indefinitely ſmall portions of time, thoſe lines may be taken for portions of the tangents D H, F I, and the triangles D C E, F C G, will be equal (95, M). But the baſes D E, F G, of equal triangles, are inverſely as the perpendiculars C H, C I. And velocities being as the baſes D E, F G, deſcribed in equal times, muſt alſo be in the ſame inverſe ratio of thoſe perpendiculars. Which was to be ſhewn.

If a body, acted on by a centripetal force, di- Q
rected to C (fig. 68), be projected from U, in a direction at right angles to U C, but with a velocity too ſmall to cauſe it to revolve in a circle U A, it will fall within the circle, by the greater prevalence of the centripetal force. As it approaches the center, its velocity muſt increaſe (96, R), and ſo muſt likewiſe its tendency to recede from the center.

center. If the centripetal force increaſes in the ſame or in a higher ratio than that tendency, the body will ſtill continue to approach, and at length fall into the center. But if the centripetal force increaſes in a leſs ratio, the increaſing velocity will cauſe the body to move in a courſe leſs and leſs inclined to the radius, till at length it becomes at right angles to it, and recedes again from the center, becauſe by the ſuppoſition, the velocity is too great for the body to move there in a circular orbit. In the receſs from the center the velocity muſt decreaſe (96, R), and a ſimilar curve be deſcribed by the body, till its courſe becomes again at right angles to the radius, and it is again cauſed to approach the center. And this alternation will continue for ever.

R The aſſertion reſpecting the ſimilarity or rather congruity of the curves between the apſides may be eaſily evinced from fig. 40. For if a body be ſuppoſed to move from H to C, and to be reflected back from C, in the direction and with the velocity C D, it is unneceſſary to ſhew that it would again deſcribe the ſame polygon C H. And the ſame holds good of curves (95, M). Now a body that arrives at its apſis muſt move with a velocity and direction which, with reſpect to the center, is equivalent to its being reflected back in the contrary direction, becauſe in either caſe it will begin to move with a given velocity in the tangent of the ſame circle.

It

It would carry us too far into the consideration S
of the nature of those curves that may be described by bodies acted on by centripetal forces, if we were to enquire minutely into the consequences that would follow from the supposition of various laws of its increase or diminution, according to the distance. Our purpose will be sufficiently answered by attending to the velocities of revolving bodies in their apsides. Let a body (fig. 68) be pro- T
jected from U towards A, in a direction at right angles to C U, a line drawn from the point C, to which let the centripetal force be supposed to be directed. Suppose the velocity of projection to be less than would be required to carry the body in a circle at U, and the body will accede towards the center, by passing through a curve U D F. If the law of the centripetal force be such (201, Q) as to suffer the body to recede again, after coming within a certain distance of the center, there will be some point L, at which the body, previous to its going off, will neither approach nor recede from the center, but move in a direction at right angles to the radius. This point is the lower apsis, and its velocity will then be inversely as the perpendicular C L (201, P).

Let us suppose the centripetal force to be in- U
versely as the cube of the distance from C. Then (200, M) the velocities necessary to carry a body in a circle at U or L will be inversely as the distances U C, L C. The actual velocities at U and L are in the same ratio, and the velocity at U is known to be

be too ſmall to carry it in a circle there. Conſequently, the actual velocity at L muſt be likewiſe too ſmall in the ſame ratio, and the body will continue to approach the center, after having paſſed the apſis. It will not, therefore, deſcribe a curve congruous with the curve deſcribed in its paſſage between the two apſides. But this laſt conſequence being contrary to what has been already proved (202, R) muſt be falſe, and ſo muſt likewiſe be the original ſuppoſition from which it was deduced.
V Conſequently it is not true, that a body projected with a velocity too ſmall to keep it in a circle, and acted on by a centripetal force inverſely as the cubes of the diſtances, can ever arrive at the lower
W apſis. It muſt therefore continually approach the center, and at length fall into it.

X If the body be ſuppoſed to be originally projected from L, the lower apſis, with a velocity too great for the centripetal force, according to the ſame law, to retain it in a circle, it may be ſhewn by ſimilar argumentation that it would never arrive at the higher apſis, but would continually recede from the center.

Y Thus it appears, that the inverted ratio of the cubes of the diſtances is the law of centripetal force that limits the revolutions of bodies in curves that admit of alternate approach and receſs from the center. For if, according to this law, a body, after once beginning to approach the center or to recede from it, cannot but continue that approach or receſs, it muſt be much more ſtrongly urged in the ſame

manner

manner by a centripetal force that follows the inverted ratio of some higher power of the distance. And again, if the law of the force follows some inverted ratio less than that of the cube of the distance, the velocities required to retain bodies in circular orbits will be less than after the inverted ratio of the distance as that law would require (200, L, M). Whence it follows, that since the velocities in descending from the upper apsis increase faster than the distances decrease, the perpendicular C H, fig. 67, being less than the distance C D (201, P), the motion of the body will be directed less and less towards the center, till it becomes at right angles to the radius, the body being then in the lower apsis. After which it must ascend through a curve similar and equal to that it before described in passing between the apsides (202, R).

Hence it is seen, that when the centripetal force increases in approaching the center in a less ratio than the inverse ratio of the cubes of the distances, the law of its increase may be found from the quantity of angular motion employed in passing from one apsis to the other. For the distance between the upper and lower apsis will be greater the nearer the law of the centripetal force approaches to that ratio, because the body must run through a greater space before the tendency to recede from the center, arising from the velocity and direction, can be equal to the centripetal force. Z

A

If a body revolves in an elliptical orbit, describing equal areas in equal times about one of the foci, B

foci, the apsides will be at the two extremities of the transverse diameter, or 180° of angular motion apart, the centripetal force will be directed to that focus (95, N), and its intensity will be inversely as the square of the distance*.

C If a body revolves between two apsides, and the centripetal force be inversely as some power of the distance, greater than the square and less than the cube, the distance between the apsides will be more than 180°. But if the centripetal force be inversely as some power of the distance less than the square, the distance between the apsides will be less than 180° (205, A). In these cases the orbit may be considered as an ellipsis whose transverse diameter, or line of the apsides, is not stationary, but revolves on the focus to which the force

D is directed. The apsides may therefore be said to revolve in consequentia, or with the moving body, when the force in approaching the center is greater than after the inverse ratio of the square of the distance; or to revolve in antecedentia when the

E force is less than after that ratio. And the quiescence of the apsides will be a proof, that the centripetal force is accurately in the inverse ratio of the square of the distance.

F The periodical time of a revolution about the focus in a quiescent ellipsis is equal to that which would be employed in describing a circle whose radius is half the transverse diameter of the ellipsis †.

* Principia, I. §§. 3. 9.

† Principia, I. 15.

Therefore,

Therefore, if the ſquares of the periodical times G of bodies revolving in ellipſes be directly as the cubes of the mean diſtances (200, O) the centripetal forces will be inverſely as the ſquares of the diſtances.

CHAP. II.

THE UNIVERSALITY OF GRAVITATION DEDUCED FROM ITS EFFECTS.

THE planetary bodies being in motion would (21, P) continue to move for ever in right lines, unleſs compelled to change their ſtate by forces impreſſed. But they move in curve lines (121, K), H and conſequently muſt be acted on by forces that continually deflect their courſes out of the right lined direction.

Every primary planet moves with ſuch a velocity I and direction, that a line joining the centers of the planet and the Sun deſcribes equal areas in equal times (121, K). Whence it follows, that the centripetal forces which retain theſe planets in their orbits are (95, N) directed to the Sun's center.

The periodical times of the primary planets are ſuch, that their ſquares are directly in proportion to the cubes of their mean diſtances from the Sun. Their orbits are (121, K) elliptical, and their apſides quieſcent. From theſe phenomena it is proved K (207, G. 205, B. 206, E), that the centripetal forces are inverſely as the ſquares of the diſtances from the Sun.

Every ſecondary planet moves with ſuch a velocity and direction, that a line joining the center of the ſecondary with that and its primary, de-
L ſcribes equal areas in equal times. The centripetal forces retaining theſe bodies in their orbits conſequently are (95, N) directed to the centers of their reſpective primaries.

The periodical times of Saturn's moons are ſuch that their ſquares are directly in proportion to the
M cubes of their diſtances. And, therefore, the centripetal forces are inverſely as the ſquares of the diſtances (200, O).

N The ſame phenomenon in Jupiter's moons ſhew, that their centripetal forces follow the ſame law.

O The orbits of Saturn's and Jupiter's moons are here taken to be circular. For obſervation has not yet eſtabliſhed the eccentricity of any of theſe orbits, except that of Jupiter's fourth ſatellite.

The Moon is carried about the Earth with ſuch a velocity and direction, that a line joining its center and that of the Earth, deſcribes equal areas
P in equal times (143, F). It is therefore retained in its orbit by a force directed to the Earth's center.

Q The Moon's orbit is elliptical, and its apſides quieſcent. Its centripetal force is therefore inverſely as the ſquare of its diſtance (205, B. 206, E).

Every comet moves with ſuch a velocity and direction, that a line joining the centers of the comet and Sun deſcribes equal areas in equal times

times (160, V). The centripetal forces retaining R
the comets in their orbits is (95, N) therefore directed to the Sun.

All the comets are obferved to defcribe either ellipfes, or parabolas, which in all probability are the lower portions of ellipfes (160, U, V). Thofe whofe return has been obferved have their apfides quiefcent. Whence it follows * (205, B. 206, E), S
that the centripetal forces are inverfely as the fquares of the diftances from the Sun.

It is not to be underftood that the planetary T
phenomena are in abfolute ftrictnefs as given in this place. But the irregularities are very fmall, and it will hereafter be feen, that they are of fuch a nature as to give additional force to the deductions here made.

The force that retains the Moon in its orbit is U
the fame with that which caufes bodies near the Earth's furface to be heavy, and is called Gravity.

To prove this important truth, let us take 57′ V
for the Moon's horizontal parallax at its mean diftance, and that diftance will, by plane trigonometry, be found to be 60.314 femidiameters of the Earth. The periodical time of the Moon is 27 d. 7 h. 43 m. (142, A), or 39343 minutes, which is the fame period as would obtain if its orbit were a circle (206, F) whofe radius is equal to the mean diftance, and the centripetal force remained unaltered. To come at the effects of this force more

* For the parabola, fee Principia, I. 13.

readily, it will be convenient to attend to this circular revolution. In one minute of time the Moon in this orbit would paſs through the $\frac{1}{39343}$ part, or an arc of 32.941 ſeconds of meaſure. The mean length of a degree on the Earth's meridian is 342516 Paris feet, which number multiplied by 60.314, will give 20658510 Paris feet for the length of a degree in the ſuppoſed circular orbit. Whence the arc of 32.941 ſeconds paſſed through in a minute may be found in feet, as alſo its verſed ſine. The verſed ſine being the ſpace through which the Moon muſt fall beneath the tangent in the time of one minute, will be the effect of, and will meaſure the centripetal force (35, E). This ſpace or verſed ſine is 15.0944 Paris feet. Now, becauſe the Moon's centripetal force is inverſely as the ſquare of the diſtance from the Earth's center (208, Q), we may find what its effect would be at the Earth's ſurface by ſaying, As the ſquare of 60.314, the Moon's diſtance from the Earth's center; Is to the ſquare of 1, the diſtance of the ſurface of the Earth from its center: So is the meaſure (35, E) of the centripetal force at the Moon 15.0944 feet; To the meaſure or effect of the ſame force at the Earth's ſurface, or 54910 feet.

W To find whether this ſpace agrees with the fall of bodies at the Earth's ſurface by gravity, we muſt reduce the time to one ſecond; becauſe we have no means of directly meaſuring the actual fall of bodies during ſo long a time as one minute. Now, though

though the centripetal force of the Moon increaſes in approaching the Earth's center, yet theſe meaſures of that force are ſtrictly accurate, becauſe conſidered as they obtain in circular orbits where the diſtance of the body is not diminiſhed by its fall: but the difference would in the preſent caſe be abſolutely inſenſible in ſo ſhort times as a ſecond or a minute, even if we ſuppoſed the falling body to be moved in a right line directed to the center. The ſpaces deſcribed by falling will conſequently be deſcribed by an uniformly accelerated motion, and will be (29, o) as the ſquares of the times. Therefore as the ſquare of 60 ſeconds; Is to the ſquare of 1 ſecond: So is 54910 feet; To 15.2528 feet that bodies would fall through at the Earth's ſurface in a ſecond by the action of the centripetal force that retains the Moon in its orbit.

But bodies fall through 15.084 Paris feet in a x ſecond, by the action of gravity. The fall of bodies near the Earth's ſurface and of the Moon are effects of the ſame kind, and therefore, by the ſecond rule of philoſophizing, are produced by the ſame cauſe. That is to ſay, the Moon is retained in its orbit by gravity.

Moreover, ſince it is eſtabliſhed from the Moon's y revolution in its orbit that a centripetal force exiſts and acts in the direction towards the center of the Earth, and no good reaſon can be given againſt its action that would take place, according to its law, on bodies any where ſituated, it muſt follow, that bodies fall near the Earth's ſurface, ei-

 ther

ther by this force alone, or by this force in conjunction with ſome other. But this latter conſequence cannot be admitted, becauſe bodies would then fall with greater velocity than the Moon at the ſame diſtance; whereas it has been juſt ſhewn, that their velocities are ſomewhat leſs.

Z And even this difference between the velocities of the Moon and of ſmaller bodies affords an additional proof, that the ſame force of gravity is concerned in both. For the force would have proved accurately the ſame, as far as obſervation can meaſure its effects, if proper allowance, according to the laws of gravity known from its effects on heavy bodies, had been made in the computation for the maſs of the Moon, and the Sun's action on the Earth and Moon.

A The revolutions of the ſatellites of Jupiter and Saturn, and alſo thoſe of the primary planets and of the comets, are phenomena of the ſame kind as the Moon's revolution about the Earth, and are therefore (6, 11) to be attributed to the ſame cauſes, namely, to an original or projectile motion compounded with a motion produced by gravity.

B Action and re-action (22, R) being equal, it follows alſo, becauſe the ſecondary planets gravitate towards their primaries, and the primary planets together with their ſatellites, as likewiſe the comets, gravitate towards the Sun, that the primary planets muſt likewiſe gravitate towards their ſecondaries, and the Sun towards the whole
C ſyſtem. That is to ſay, gravitation is univerſal, or a property of all bodies whatſoever.

The

The mutual action of two bodies (22, R. 95, O) D
that gravitate to each other is the caufe that if they fall, both will approach the common center of gravity with equal quantities of motion, or if they have at the fame time a projectile velocity, both will abfolutely revolve about that center. If one of the two bodies exceed the other indefinitely in mafs, its velocity (19, L) of approach will be indefinitely lefs than that of the other in the former cafe, or the radius of its orbit will be indefinitely lefs than that of the other in the latter cafe. Whence the whole relative velocity of ap- E
proach may be taken for the abfolute velocity produced in the lefs body; or the orbit defcribed by the lefs body about the greater may be taken for that which it defcribes about the common center of gravity.

The fall of bodies near the Earth's furface may F
be regarded as their abfolute motion. For the magnitude of the Earth is fo great with refpect to the bodies with which art can make experiments, that its velocity is incomparably fmaller than the differences which the imperfections of the fenfes muft caufe in all obfervations. We may G
likewife in this place confider the motions of the planets and comets about the Sun as abfolute, though the magnitude of that luminary is not fo exceffive as to render thofe of the planets inconfiderable. The fame is to be underftood of the H
fatellites of Jupiter and Saturn.

All bodies fall near the Earth's furface with I
equal velocities (26, A, B). The planets and comets being accelerated towards the Sun by powers which

are (207, K. 209, S) inverfely as the fquares of the diftances, would confequently be equally accelerated at equal diftances. The fame is true of the fatellites with refpect to the Sun, becaufe they revolve together with their primaries in the annual orbits, and alfo with refpect to their primaries (208, M, N).

K From thefe equal accelerations it follows, that the force of gravity which urges minute bodies towards any other larger body, is in proportion to the mafs of the body urged.

L This law of gravity being eftablifhed from obfervations in cafes where the velocity of the fmaller body can be taken without fenfible error, we may again refume the confideration of the re-action of the fmaller. Let us call the larger L, and the fmaller S. Then, becaufe S is urged towards L, by a force which is as the magnitude of S, L will be urged (22, R) towards S, with the fame force. That is, if S becomes larger, it will attract L the more ftrongly in proportion. Now, S may be imagined to become larger, fo as even to exceed L in any ratio whatever, and the increafe of at-
M traction will ftill obtain. Therefore, a given body not only attracts another, in proportion to the mafs of this laft, but alfo in proportion to its own
N mafs. That is to fay, the force of gravitation exerted between two bodies is in the compound ratio of their maffes.

O The abfolute force of gravity being in the compound ratio of the maffes of the two bodies, or as the maffes multiplied into each other, will be meafured

meaſured by the quantity of motion produced in either body (21, Q. 19, L) in a given time, or by the maſs of the body multiplied into its velocity; that is, the numeral product of the maſſes will be equal to the numeral product of the maſs of one of the bodies into its velocity. Now, if each of theſe equal products be divided by the maſs of the body, whoſe velocity is conſidered, the two remainders will be equal, namely, the maſs of the one body will be equal to the velocity of the other. Or, more P clearly (becauſe we have uſed number, in order to avoid the compariſon of ratios, which is leſs generally underſtood) the maſs of one body will be as the velocity or acceleration of the other. Whence Q it follows, that if the velocities produced in bodies by gravitation be known, the proportional maſſes of the bodies towards which they are urged will be alſo known.

From this conſequence the maſs of any large pla- R net may be known from the velocity of deſcent it produces in bodies indefinitely ſmaller than itſelf. For the relative velocities of ſuch ſmall bodies (213, E) with reſpect to the larger, may be taken for their abſolute velocities. Now, the velocities of the planets towards the Sun, of the ſatellites of Jupiter and Saturn towards their reſpective primaries (213, G), and of projectiles near the Earth's ſurface, being reduced to equal diſtances of the attracting bodies, as was done in comparing the Moon's gravity with that of terreſtrial bodies (209, V, W, X) or otherwiſe, are, and conſequently likewiſe their maſſes are, (nearly)

 as

as the numbers 1, $\frac{1}{1067}$, $\frac{1}{3021}$, and $\frac{1}{169282}$. Hence alſo the denſities may be found, becauſe their bulks
T are known from obſervation. Thus, the denſities, or ſpecific gravities, are in the four bodies juſt mentioned, 100, 94½, 36, and 400. Theſe numbers may be familiarized to the imagination, by obſerving, that if the mean denſity of the Earth be ſuppoſed to anſwer to that of common green glaſs, the Sun's denſity would be equal to that of dry pear-tree, Jupiter's to cedar, and Saturn's to cork.

U Our knowledge of the remote parts of the planetary ſyſtem is too imperfect to admit of many re-
V marks on the facts we can obſerve and deduce. It is, however, worthy of notice, that the immenſely large planets, Jupiter, Saturn, and the Georgium Sidus, would have occaſioned great irregularities by their attractions on the other bodies of the ſyſtem; if, inſtead of being placed at the great diſtances they are from the common center of gravity, and from each other, they had occupied the places of the ſmall planets, Mars, the Earth, Venus or Mer-
W cury. And if the celeſtial ſpaces be not abſolutely vacuous, but poſſeſſed by ſome very rare matter, (175, A) the reſiſtance ſuch matter muſt afford in the courſe of ages to the motions of the planets, will be brought nearly to equality in its effects, if the planets which move ſwifter are at the ſame time more denſe. Laſtly, becauſe it is obſerved in the conſtitution of terreſtrial bodies, that the denſer require, in many inſtances, a higher degree of heat to produce given changes in them, it has been conjectured,

jectured, not without ſome degree of probability, that the planets nearer the Sun are for this purpoſe formed of denſer materials adapted to their ſituation.

From the univerſality of gravitation (212, c) x
it is deduced, that the fixed ſtars are either falling towards the common center of gravity of the univerſe, or are made to deſcribe immenſe orbits which ultimately reſpect that center. The life of man, aſſiſted by every traditional record for thouſands of years, ſeems too ſhort to aſcertain the reſult of this ſublime enquiry. It will be a grand acquiſition, if the repeated obſervations of ſeveral centuries to come ſhould determine the proper motions of the vaſt number of ſuns that compoſe the nebula, (194, z) of which our whole planetary ſyſtem, with all its comets, forms ſo inconſiderable a part.

CHAP.

CHAP. III.

OF THE IRREGULARITIES ARISING FROM THE MUTUAL GRAVITATION OF THE PLANETS.

Y IF the Sun were at reſt, and the planets did not mutually gravitate towards each other, they would deſcribe ellipſes, having the Sun in the common focus. But ſince they mutually act on the Sun, and on each other (212, c) it muſt follow that the Sun is perpetually moved about the center of gravity of all the planets, which center is the common focus of their orbits. This center, by reaſon of the Sun's very great bulk, can, in no ſituation, exceed the diſtance of its ſemidiameter from its ſurface.
Z Some ſmall irregularities ariſe from theſe mutual actions, but much leſs than would enſue if the Sun were at reſt, or not ſubject to the re-action of the other planets. The irregularities in the motions of the primary planets are ſcarcely conſiderable enough to come under obſervation in the courſe of many revolutions: thoſe of the Moon, on account of its nearneſs to us, and from other cauſes, have ever been ſufficiently great, to embarraſs the aſtronomical world. It will therefore be ſufficient to explain the latter, and apply the explanation to the former, which are effects of the ſame kind.

H If the actions of the Sun upon the Earth and Moon were equal upon each, according to their maſſes, and tended to produce motions in parallel directions,

directions, their relative motions would be the same as if no such forces acted upon them (79, W). But these forces vary, both in quantity and direction, according to the various relative situations of the Earth and Moon.

Let the point S (fig. 66) represent the Sun, E the Earth, and ADBC the orbit of the Moon. Then, if the Moon be at the quadrature A, the distances ES and AS of the Earth and Moon from the Sun, being equal, their gravities towards S will also be equal, and may be represented by those lines ES and AS. Draw the line AL parallel and equal to ES, and join LS, which will be parallel to AE. The force AS may be resolved (23, T) into the two forces AL and AE; of which AL, by reason of its parallelism and equality to ES (79, W) will not disturb their relative motions or situation: but the force AF, conspiring with that of gravity, will cause the Moon to fall farther below the tangent of its orbit than it would have done if no such disturbing force had existed. Therefore, at or near the quadratures, the Moon's gravity towards the earth is increased more than according to the regular course, and its orbit is rendered more curve. B

When the Moon is at the conjunction C, the distances ES and CS not being equal, the Moon's gravitation towards the Sun exceeds that of the Earth in the same proportion as the square of ES exceeds the square of CS. And because the excess acts contrary to the direction of the Moon's gravity towards the Earth, it diminishes the effect thereof, and

causes

cauſes the Moon to fall leſs below the tangent of its orbit than it would if no ſuch diſturbing force exiſted. A like, and very nearly equal, effect follows, when the Moon is at the oppoſition D, by the Earth's gravitation towards the Sun being greater than that of the Moon: whence their mutual gra-
E vity is diminiſhed as in the former caſe. Therefore, at or near the conjunction or oppoſition, the Moon's gravity is diminiſhed, and its orbit is rendered leſs curve.

F It is found, that the force added to the Moon's gravity at the quadratures, is to the gravity with which it would revolve about the earth in a circle at its preſent mean diſtance, if the Sun had no effect on its motion, as 1 to 190; and that the force ſubducted from its gravity at the conjunction or oppo-
G ſition is about double this quantity. The influence of the Sun, then, on the whole, increaſes the Moon's diſtance from the Earth, and augments its periodical time; and ſince this influence is moſt conſiderable when the Earth is neareſt the Sun, or in its perihelium, its periodical time muſt then be the greateſt, as appears likewiſe from obſervation (144, K).

H To ſhew the effect of the Sun in diſturbing the Moon's motion at any ſituation between the conjunction and one of the quadratures, ſuppoſe at M (fig. 66) let ES repreſent the Earth's gravity towards the Sun; draw the line MS, which continue towards G; from M ſet off MG, ſo that MG may be to ES as the ſquare of the Earth's diſtance ES is to

the

the square of the Moon's distance M S; and M G will represent the Moon's gravity towards the Sun. From M draw M F parallel, and equal to E S; join F G, and draw M H parallel, and equal to F G. The force M G may be resolved into M F and M H; of which M F, by reason of its parallelism and equality to E S, will not disturb the relative motions or situations of the Moon and Earth; M H then is the disturbing force. Draw the tangent M K to the Moon's orbit, and continue the radius E M towards I; draw H I parallel to K M, and intersecting M I in I, and complete the parallelogram by drawing H K parallel to I M, and intersecting M K in K. The force M H may be resolved into M I and M K; of which M I affects the gravity, and M K the velocity of the Moon. When the force M H coincides with the tangent, that is, when the Moon is 35° 16′ distant from the quadrature, the force M I, which affects the gravity, vanishes; and when the force M H coincides with the radius, that is, when the Moon is either in the conjunction or quadrature, the force M K vanishes. Between the quadrature and the distance of 35° 16′ from it, the line or force M H falls within the tangent, and consequently the force M I is directed towards E, and the Moon's gravity is increased: but, at any greater distance from the quadrature, the line M H falls without the tangent, and the force M I is directed from E, the Moon's gravity being diminished. It is evident that the force M K is always directed to some point in the line which passes through the centers of the Sun and Earth; therefore

therefore it will accelerate the Moon's motion, while it is approaching towards that line, or the conjunction, and similarly retard it as it recedes from it, or approaches towards the quadrature, by conspiring with the motion in one case, and subducting from it in the other.

K As the Moon's gravity towards the Sun at the conjunction is diminished by a quantity which is as the difference of the squares of their distances; and as this difference, on account of the very great distance of the Sun, is nearly the same when the Moon is at the opposition, the mutual tendency to separate, or diminution of gravity, will be very nearly the same. Whence it easily follows, that all the irregularities which have been explained as happening between the quadratures and conjunction must in like circumstances take place between the quadratures and the opposition.

L If the Moon revolved about the Earth in a circular orbit, the Sun's disturbing influence being supposed not to act, then this influence being supposed to act would convert the orbit into an ellipsis. For the increase of gravity renders it more curve at the quadratures, by causing the Moon to fall further below the tangent; and the diminution of gravity, as well as the increasing velocity, renders the orbit less curve at the conjunction and opposition, by causing the Moon to fall less below the tangent in a given time. Therefore an ellipsis would be described, whose less or more convex parts would be at the quadratures, and whose longest diameter would

would paſs through them. Conſequently the Moon would be fartheſt from the Earth at the quadratures, and neareſt at the conjunction and oppoſition. Neither is it ſtrange that the Moon ſhould approach or come nearer to the Earth at the time when its gravity is the leaſt, ſince that approach is not the immediate conſequence of the decreaſe of gravity, but of the curvity of its orbit near the quadratures; and in like manner, its receſs from the Earth at the quadratures does not ariſe immediately from its diminiſhed gravity, but from the velocity and direction acquired at the conjunction or oppoſition.

But as the Moon's orbit is, independent of the Sun's action, an ellipſis, theſe effects take place only as far as circumſtances permit. M

The Moon's gravity towards the Earth being thus ſubject to a continual change in its ratio, its orbit is of no conſtant form. The law of its gravity being nearly in the inverſe proportion of the ſquares of the diſtances, its orbit is nearly a quieſcent ellipſis (206); but the deviations from this law occaſions its apſides to move direct or retrograde, according as thoſe deviations are in defect or exceſs (206, C). Aſtronomers, to reduce the motion of the apſides to computation, ſuppoſe the revolving body to move in an ellipſis, whoſe tranſverſe diameter or line of the apſides revolves at the ſame time about the focus of the orbit. When the Moon is in the conjunction or oppoſition, the Sun ſubducts from its gravity (220, E), and that the more the greater its diſtance is from the Earth, ſo that N

that its gravity follows a greater proportion than the inverted ratio of the ſquare of the diſtance, and conſequently the apſides of its orbit muſt then move in conſequentia, or direct (206, D). In the quadratures the Sun adds to the Moon's gravity (219, G), and that the more the greater its diſtance from the Earth, ſo that its gravity follows a leſs proportion than the inverted ratio of the ſquare of her diſtance, and conſequently the apſides of its orbit muſt then move in antecedentia, or retro-
O grade (206, E). But becauſe the action of the Sun ſubducts more from the Moon's gravity in the conjunction and oppoſition than it adds to it in the quadratures (220, F) the direct motion exceeds the retrograde, and at the end of each revolution the apſides are found to be advanced according to the order of the ſigns.

P If the plane of the Moon's orbit coincided with that of the ecliptic, theſe would be the only irregularities ariſing from the Sun's action; but becauſe it is inclined to the plane of the ecliptic in an angle of about five degrees, the whole diſturbing force does not act upon the Moon's motion in its orbit, a ſmall part of the force being employed to draw it out of the plane of the orbit into that of the ecliptic.

Q Of the forces MK and MI, fig. 66, which diſturb the Moon's motion, MI being always in the direction of the radius, can have no effect in drawing it out of the plane of its orbit. And if the force MK really coincided with the tangent, as we, neglecting

lecting the ſmall deviation ariſing from the obliquity of the Moon's orbit have hitherto ſuppoſed, it is evident that its only effect would be that of accelerating or retarding the Moon's motion, without affecting the plane of its orbit. But becauſe that force is always directed to ſome point in the line which paſſes through the centers of the Sun and Earth (221, 1) it is evident that it can coincide with the tangent only when that line is in the plane of the Moon's orbit; that is to ſay, when the nodes are in the conjunction and oppoſition. At all other times the force M K muſt decline to the northward or ſouthward of the tangent, and compounding itſelf with the Moon's motion, will not only accelerate or retard it, according to the circumſtances before explained, but will likewiſe alter its direction, deflecting it towards that ſide of the orbit on which the point, the force M K, tends to, is ſituated. This deflection cauſes the Moon to arrive at the ecliptic either ſooner or later than it would otherwiſe have done; or, in other words, it occaſions the interſection of its orbit with the ecliptic to happen in a point of the ecliptic, either nearer to, or further from, the Moon, than that in which it would have happened if ſuch deflection had not taken place.

To illuſtrate this, let the elliptical projection R C O Q N (fig. 67) repreſent a circle in the plane of the ecliptic, M O P N the Moon's orbit, interſecting the ecliptic in the nodes N and O. Suppoſe the Moon to be in the northern part of its orbit at M, and moving towards the node O; the diſturbing force

M K, which tends towards a point in the line S E to the ſouthward of the tangent M T, will be compounded with the tangental force, and will cauſe the Moon to deſcribe the arc M m, to which M R is tangent, inſtead of the arc M O; whence the node O is ſaid to be moved to m. In this manner the motion of the nodes may be explained for any other ſituation.

S This motion evidently depends on a twofold circumſtance, namely, the quantity and direction of the force M K. If the force M K be increaſed, its direction remaining the ſame, it will deflect the curve of the Moon's path from its orbit in a greater degree; and on the other hand, if its direction be altered, ſo as to approach nearer to a right angle with the tangent, it will cauſe a greater deflection,
T though its quantity remain the ſame. When the Moon is in the quadratures, the force M K vaniſhes, (220, H) conſequently the nodes are then ſtationary. When the Moon is at the octant, or forty-five degrees from the quadrature, the force M K is greateſt of all, and therefore the motion of the nodes is then moſt conſiderable, as far as it
U depends on the quantity of M K. But the direction of this force in like circumſtances depends on the ſituation of the line of the nodes. If the line of the nodes coincides with the line paſſing through the centers of the Sun and Earth, the force M K coincides with the tangent of the Moon's orbit, and the nodes are ſtationary. And the farther the node is removed from that line, the farther is that

line

line removed from the plane of the Moon's orbit; till the line of the nodes is in the quadratures, at which time the line paſſing through the centers of the Sun and Earth, makes an angle with the plane of the Moon's orbit equal to its whole inclination, or five degrees: conſequently the angle formed between M K and the tangent in like circumſtances is then greateſt, M K being directed to a point in a line which is further from the plane of the Moon's orbit than at any other time, and of courſe the motion of the nodes is then moſt conſiderable.

To determine the quantity and direction of the motion of the nodes, ſuppoſe the Moon in the quarter preceding the conjunction, and the node towards which it is moving to be between it and the conjunction: in this caſe its motion is directed to a point in the ecliptic, which is leſs diſtant than the point towards which the force M K is directed: the force M K then, compounding with the Moon's motion, cauſes it to be directed to a point more diſtant than it would otherwiſe have been; that is to ſay, the node, towards which the Moon moves, is moved towards the conjunction. When the Moon has paſſed the node, its courſe is directed to the other node, which is a point in the ecliptic more diſtant than the point to which M K is directed, and therefore M K, compounding with its motion, cauſes it to be directed to a point leſs diſtant than it would otherwiſe have been; ſo that in this caſe likewiſe, the enſuing node is moved towards the conjunction. After the Moon has paſſed

the conjunction, the force M K ſtill continues to deflect its courſe towards the ecliptic, and conſequently the motion of the node is the ſame way till its arrival at the quadrature. Suppoſe again, the Moon to be at the conjunction, and the node towards which it is moving to be between it and the quadrature. In this caſe the force M K compounding with the Moon's motion, cauſes it to move towards a point in the ecliptic leſs diſtant than it would otherwiſe have done, ſo that the enſuing node is brought towards the conjunction. When the Moon has paſſed the node, the force M K ſtill continuing to deflect its courſe towards the ſame ſide of its orbit, produces a contrary effect, namely, as it before occaſioned it to converge to the ecliptic, ſo it now cauſes it to diverge from it, and its motion in conſequence tends continually to a point in the ecliptic more diſtant than it would otherwiſe have done: the enſuing node in this inſtance being alſo brought towards the conjunction.

w As the diſturbing forces are very nearly the ſame in the half of the Moon's orbit, (222, K) which is fartheſt from the Sun, this laſt paragraph is true, when it moves in that part of its orbit, if the word oppoſition be every where inſerted inſtead of the word conjunction.

x Whence it is eaſy to deduce this general rule, that when the Moon is in the part of its orbit neareſt the Sun, the node towards which it is moving is made to move towards the conjunction: and when it is in the part of its orbit fartheſt from the Sun, the node

node towards which it is moving is made to move towards the oppoſition.

Suppoſe the Moon at Q, (fig. 68) or the quadrature preceding the conjunction, then the enſuing node, if at 90° diſtance, or at the conjunction C, will be ſtationary (226, U), but if it be at a greater or leſs diſtance, it will be brought towards C (228, X). Thus, if the nodes be in the poſition M N, the enſuing node M, being at a leſs diſtance from Q than 90°, will move towards C, or direct, while the Moon moves through the arc Q M; after which N becomes the enſuing node, and likewiſe moves towards the conjunction C, or retrograde during the Moon's motion through the arc M R. And becauſe the arc M R exceeds Q M, the retrograde motion exceeds the direct. Again, if the nodes be in the poſition n m, the enſuing node n being at a greater diſtance from Q than 90°, will move towards C, or retrograde, during the Moon's motion through the arc Q n; after which the node m becomes the enſuing node, and likewiſe moves towards the conjunction C, or direct, during the Moon's motion through the arc n R. And becauſe the arc Q n exceeds N R, the retrograde motion here alſo exceeds the direct. If the nodes be in the quadratures Q R, the enſuing node R removes towards C, or retrograde, during the Moon's motion through the arc Q R, or almoſt the whole ſemi-orbit. The ſame may be ſhewn in the other half of the orbit R O Q with reſpect to the oppoſition O; and therefore, in every revolution of the Moon, the retrograde motion of the nodes ex- Y

Z ceeds the direct; and, on the whole, the nodes are carried round, contrary of the order of the ſigns.

A The line of the conjunction is by the Earth's annual motion brought into every poſſible ſituation with reſpect to the nodes in the courſe of a year, independant of their own proper motion; which laſt occaſions the change of ſituation to be performed in about nineteen days leſs.

B The inclination of the Moon's orbit being the angle which its courſe makes with the plane of the ecliptic, it is evident from what has been ſaid, that this angle is almoſt continually changing. Suppoſe the line of the nodes, by its retrograde motion, to leave the conjunction C, fig. 69, and become in the ſecond and fourth quarters as in the poſition M N, and the Moon to move from the node M to the node N; then, becauſe the enſuing node N moves (228, X) towards the conjunction C, while the Moon is in the nearer half of its orbit, the Moon's courſe muſt be continually more and more inflected towards the ecliptic, till its arrival at R. This inflection in the firſt 90°, or M A from M, prevents its diverging ſo much from the ecliptic as it would otherwiſe have done; that is to ſay, it diminiſhes the angle of the Moon's inclination. From A to R its courſe begins to converge towards the ecliptic, and this convergence is increaſed by the inflection which in the preceding 90° prevented its divergence: in the arc A R then the inclination is increaſed. During the Moon's motion from R to N, the node is moved towards the oppoſition

poſition o, and conſequently the angle of its courſe
to N is rendered leſs than it would have been if the
node has not moved; or, in other words, the in-
clination is diminiſhed. And becauſe the arc M A
added to the arc R N is greater than the arc A R, the
inclination at the ſubſequent node is leſs than at
the precedent node; and the ſame may be ſhewn
in the other half revolution N Q M. Therefore, C
while the nodes are moving from the conjunction
and oppoſition to the quadratures, the inclination
of the Moon's orbit, on the whole, diminiſhes in
every revolution till they arrive in the quadratures,
at which time it is leaſt of all. When the line of D
the nodes has paſſed the quadratures, and is in the
firſt and third quarters, as in the poſition m n, it
is eaſily ſhewn by the ſame kind of argument, that
the inclination is increaſed while the Moon paſſes
from m to Q, then diminiſhes for the remainder
of the firſt 90° or Q a, and is afterwards increaſed for
the other 90° or a n: and the ſame may be proved
for the other half revolution n R M. Conſequently, E
while the nodes are moving from the quadratures
to the conjunction and oppoſition, the inclination is
increaſed by the ſame degrees as it before was di-
miniſhed, till they arrive at the conjunction and
oppoſition, at which time it returns to its firſt quan-
tity, being then greateſt of all.

The line of the nodes in the courſe of one en- F
tire revolution, with reſpect to the Sun, is twice
in the quadratures and twice in the conjunction and
oppoſition. Therefore, the inclination of the

Moon's orbit to the ecliptic is diminiſhed and increaſed by turns, twice in every revolution of the nodes.

G All the irregularities of the Moon's motion are a little greater when in the half of its orbit neareſt the Sun, than when it is in the other half; the chief reaſon of which is, that the difference between the ſquares of the Moon's and Earth's diſtances from the Sun is greater, in proportion to the ſquares themſelves, in the former than in the latter caſe at equal elongations from the quadrature, and conſequently the diſturbing forces muſt be more conſiderable.

H Although the Moon in reality revolves about the common center of gravity between itſelf and the Earth, and not about the Earth itſelf, and conſequently their motions and irregularities are ſimilar, and not confined to the Moon alone; yet it may be eaſily conceived, that the concluſions are not affected in any degree that may be here regarded, when, for the ſake of conciſeneſs, we ſuppoſe one of the two bodies to be quieſcent, and the other to revolve about it.

I Irregularities of the ſame kind take place among the primary planets by their mutual actions on each other, but the quantities are not conſiderable. Hence the apſides of the planets are found to move (224, o) in conſequentia, but ſo very ſlowly, that ſome have doubted whether they move at all.

K The motions of the aphelia of Saturn, Jupiter, Mars, the Earth, Venus, and Mercury, as deduced from the

compariſon

comparison of distant observations, are respectively, 2° 30′, 1° 43′ 20″, 1° 51′ 40″, 1° 49′ 10″, 4° 10′, 1° 57′ 40″, in a century. The actions L of the inferior planets on each other are very minute, on account of the smallness of their bulks; but those of Jupiter and Saturn are not altogether insensible. When Jupiter is between the Sun and M Saturn, its whole attraction acts upon Saturn, and increases the gravity of that planet towards the Sun. This is found, by comparing the respective masses of Jupiter and the Sun, and the respective squares of their distances from Saturn to be equal to $\frac{1}{221}$ of the Sun's action upon Saturn. Saturn, N on the other hand, at the conjunction, acts upon Jupiter and the Sun in the same direction, and therefore disturbs their relative position only so far as its actions on each are not equal. The difference of these actions is found by the same principles to be $\frac{1}{1923}$ of Jupiter's whole gravity.

CHAP.

CHAP. IV.

OF THE FIGURES OF THE PLANETS; THE PRECESSION OF THE EQUINOXES, AND THE NUTATION OF THE EARTH'S AXIS.

o A MASS of fluid matter will, by its gravity, form itſelf into a ſphere. For if the whole maſs be conceived to be divided into a number of ſimilar pyramids or columns, terminating in the center of gravity, and one of theſe columns be longer or higher than the reſt, its projecting part will ſpread ſideways over the other columns, till the heights are all equal to its own. The ſame is true of any other eminences or longer columns. Therefore, when all the ſubſidences are effected, and the maſs is at reſt, its form will be that of a ſolid, whoſe ſurface is every where equidiſtant from its center. And this is the property of a ſphere.

p This takes place in a maſs whoſe parts preſerve the ſame ſituation with reſpect to its center; but if the ſphere be cauſed to revolve on its own axis, a centrifugal force will be produced that will diminiſh the gravity of all its parts, except thoſe which are ſituated in the axis of rotation. This diminution will be greateſt in the equator, becauſe the velocity is there greateſt, and becauſe the centrifugal force acts directly againſt the force of gravity. And the nearer any parallel of latitude or circle of rotation is to one of the poles, the leſs will

will the gravity of the parts be affected, both the above mentioned causes being less. The equilibrium, before subsisting between the columns in a spherical figure, will consequently be destroyed, and the same effect must take place, as would have followed if the columns at the polar regions had been lengthened or augmented in mass beyond those near the equatorial parts, that is to say, the columns near the poles will spread over those towards the equator, till the difference of their lengths compensates for the difference of their gravities. Thus the sphere, by its rotation, will be changed into a solid, whose radii, drawn to the center, are longest near the equator, and shortest towards the poles, the axis being the shortest of all its diameters. Q

By computation grounded on these and other considerations, it is shewn, that bodies at the equator of the Earth lose more than $\frac{1}{210}$ part of their gravity, and that the equatorial diameter is to the axis as 231 to 230, upon the supposition that the Earth is every where of the same uniform density. For what has been said of a fluid mass will hold good of the Earth, since if it were not of this figure, but spherical, the ocean would overflow the regions near the equator, and leave the polar regions elevated many miles above the level of the sea. But experience shews, that the land is in general no more elevated above the sea in one part of the globe than another. R

This decrease of gravity towards the equator is remarkably seen in the motion of pendulums. T

For

For a pendulum, which in a higher latitude vibrates ſeconds, is found to go ſlower at the equator, and that in a much greater proportion than can ariſe from the lengthening of the rod by heat, nay, even in the coldeſt parts of the mountains of Spaniſh America, which are conſtantly covered
u with ſnow. From the juſt mentioned quantity of diminution of gravity, it is not difficult to compute the length of a pendulum (87, Y) which ſhall vibrate ſeconds in a given latitude, and from the agreement of theſe computations with experience, the oblate ſpheroidal figure of the Earth, as alſo the diurnal rotation from which it originates, are both confirmed.

v The ſame concluſion has likewiſe been obtained from the labours of many ingenious and learned men, who have actually meaſured the lengths of certain portions of the meridian in different latitudes, by which it appears, that the degrees are ſhorter towards the equator than nearer the poles. Whence it follows, that the meridian is more curved near the equator, and leſs near the poles, or in other words, that the Earth is flattened about the polar regions.

w The meaſure of a degree of the meridian, beginning at the equator, was found to be 56750 French toiſes, and the meaſure of a degree of the meridian cutting the arctic circle, was found to be 57422 French toiſes *.

* See De la Lande's Aſtronomie, § 2655, & ſeq. for a detail of the principal enterpriſes on this intereſting ſubject.

Theſe

Theſe menſurations conſtitute the experimental X
proof of the Earth's rotation on its axis; for it is evident, that a centrifugal force cannot be produced but by an abſolute motion: and as the effects of this force are obſerved in the figure of the Earth, and not at all in the heavens, the motion of the Earth muſt be abſolute and real, and that of the heavens only relative and apparent.

The planet Jupiter revolves on its axis in leſs Y
than ten hours; a rapidity which much exceeds that of the Earth; and its figure differs accordingly much more from that of a ſphere, its equatorial diameter exceeding its polar diameter, according to the obſervations of aſtronomers, as 13 to 14 (126).

It has alſo been already noticed, that a ſimilar Z
phenomenon is ſeen in the planet Mars (126).

If a number of fluid bodies revolved about the A
Earth at equal diſtances from its center, they would, by the action of the Sun, or any other planet, be ſubject to irregularities of the ſame kind as the Moon has been ſhewn to have in its motion; that is, they would approach nearer, and move ſwifter at the conjunction and oppoſition than
at the quadratures (221, I. 222, L), And if the B
number of bodies were ſo great as to become contiguous, and form a fluid ring or circle, the parts of this ring would be affected in the ſame manner. If it were inclined to the ecliptic, the nodes would be ſtationary when in the conjunction or oppoſition (226, U) and be carried in a retrograde direc-

tion in the other revolutions (229, z), but most swiftly when they were situated in the quadratures (226, u). Its inclination would likewise vary in every revolution (230, B), and in a space of time somewhat less than a periodical year would be diminished and increased, by turns, twice (230, A. 231, F).

C Suppose this fluid ring to be of the same diameter as the Earth, to be placed in a cavity hollowed round the Earth at the equator, and to revolve in the same time and direction as the Earth does on its axis. Its motion would not then be uniform (237, B), but at the conjunction and opposition swifter than the surface of the Earth, and slower at the quadratures; consequently, with respect to the surface of the Earth, it would ebb and flow like a sea. For, by reason of the increased swiftness at the conjunction and opposition, and the retardation at the quadratures, the fluid, between the conjunction or opposition and the ensuing quadrature, would form a cumulus or heap, while a correspondent defect would happen in the other quadrants preceding the conjunction and opposition.

D If this ring be now supposed to be frozen or converted into a solid, the flux and reflux will cease, but the precession of the nodes and the libratory increase and decrease of the inclination will remain (237, B). Suppose the ring to adhere to the surface of the Earth at the equator, instead of being admitted into a cavity; it will then communicate part of its motion to the Earth, the nodes of whose

whoſe equator will recede, but with a much ſlower motion than thoſe of the ring would have receded, if it had not adhered to the Earth; and the obliquity or angle which the equator makes with the ecliptic, will be diminiſhed and increaſed alternately twice in a year.

The elevation of the equatorial parts of the E
Earth have the ſame effect as ſuch a ring would have; for the exceſs of matter in thoſe regions ſupplies its place.

Aſtronomers begin the year in the Spring, when F
the Sun is in that node of the equator, or equinoctial point at which the days begin to lengthen in the northern hemiſphere. Now it is plain (187), that if the equinoctial points had no motion, the Earth would complete one revolution in its orbit in the ſame time that the Sun employs in apparently paſſing from one of the equinoxes, and returning again to the ſame. But, becauſe of the retrograde motion, the line of the nodes of the equator, or diameter of the Earth which joins the equinoctial points, is brought to coincide again with the line which joins the centers of the Sun and Earth, before its periodical revolution is completed; and therefore the circle of the ſeaſons is performed in leſs time than the Earth's revolution
in its orbit. The actions both of the Sun and G
Moon on the redundant matter in the equatorial regions tend to produce this motion, which is ſo ſlow, that a complete revolution will not be finiſhed in leſs than twenty-five thouſand years.

H This is called the preceſſion of the equinoxes, and is the reaſon that the fixed ſtars appear to advance in longitude about 50 ſeconds of meaſure in a year; whence it has happened, that ſince the time of Ptolemy, the zodiacal figures have advanced the greateſt part of a whole ſign: the conſtellation Aries being ſituate in that part of the ecliptic which is denominated from Taurus, Taurus in the

I place of Gemini, &c. The difference between the natural year or period of the ſeaſons, and the periodical year, or time of the Earth's revolution in its orbit, is $26^{m}\ 34\frac{1}{2}^{s}$; for the natural year conſiſts of $365^{d}\ 5^{h}\ 48'\ 45\frac{1}{2}''$, and the periodical year of $365^{d}\ 6^{h}\ 15'\ 20''$.

K The ſideral year, or time employed by the Sun in returning to the ſame apparent poſition with reſpect to a fixed ſtar, is $365^{d}\ 6^{h}\ 9'\ 1''\frac{1}{4}$. The difference between the periodical and ſideral year is occaſioned by the motion of the apſis of the Earth's orbit (216, K).

L The libratory variation of the inclination of the equator to the ecliptic is termed the nutation of the Earth's axis. The theory of attraction had aſcertained its exiſtence, long before aſtronomical obſervations were brought to a ſufficient degree of perfection to render it ſenſible. Its whole effect ſcarcely amounts to 18 ſeconds. It was firſt obſerved by Dr. Bradley *.

* Phil. Tranſ. January, 1748.

CHAP.

CHAP. V.

OF THE TIDES.

Though the cauſe of the tides may be collected from what was ſaid in the laſt chapter; yet, as it is the only obvious inſtance we have of the mutual gravitation of the celeſtial bodies, it will be proper to give a more particular explanation of it. M

If the Earth were every where covered with a deep ſea, it is plain, from the reaſons before recited (238, c), that the water would not, in the diurnal rotation, move with the ſame uniform velocity as the Earth. For, if the apparent diurnal revolution of the Moon be called a lunar day, and be divided into twenty-four equal parts or hours, the water ſituated near the meridian over which the Moon at any time is, will move ſwifter, and the water ſituated near the meridian ſix hours to the eaſtward or weſtward, will move ſlower: becauſe the water on each parallel of latitude may be conceived to be a fluid ring, and will be affected by the diſturbing force nearly in proportion to its diameter. The ſea, then, being accelerated at the meridian upon which the Moon is, and retarded at the meridian, that is 90° or 6 hours to the eaſtward, will be accumulated between the two places; its greateſt height being at the half N

distance, or meridian which the Moon has passed
three hours. And on the other hand, the retarda-
tion at the quadrature to the westward, preventing
the water from flowing as fast as the acceleration,
at the meridian at which the Moon is, carries it
away, the sea must of course be depressed between
the two places, its greatest depression being at the
half distance, or meridian at which the Moon will
arrive in three hours. A similar accumulation and
diminution will happen at the places which are
diametrically opposite to those here described,
o though not altogether so great (232, o). The
disturbing force of the Sun will act in a like man-
ner, but less strongly; for, though the Moon's at-
tractive force be vastly less than that of the Sun,
yet, because its distance in comparison to that of
the Sun from the Earth is very small, the forces
with which it acts on different parts of the Earth
will vary more considerably from parallelism and
equality; and the irregularities in any system, which
arise from the actions of forces from without, are
occasioned (79, w), not by the whole actions of
the forces, but only by their differences in quantity,
or want of parallelism in direction.

p Thus, it is evident, that the sea, as far as cir-
cumstances allow, must in every place be raised
to its greatest and least height, alternately twice in
each lunar day. Being elevated once at three lunar
hours after the Moon has passed the meridian of
the place, and once at twelve hours after, or three
hours after the Moon has passed the opposite part
of

of the same meridian; and at six hours after each of these elevations its greatest depressions follow. This appears by the tides in the Atlantic ocean on O
the western coasts of Europe and Africa, and in the Pacific ocean on the open coasts of Asia and America, where high-water always happens about the third hour after the Moon has passed the meridian, except where the motion of the sea is somewhat impeded by flats or shoals.

The effects of the disturbing forces of the Sun R
and Moon are not seen distinctly, but compounding with each other produce a motion which is different from what would have arisen from the single action of either luminary. At the time of the conjunc- S
tion or opposition their effects are united, and the tides are greatest, being what are called Spring-tides. When the Moon is in the quadrature, the T
Sun's action raises the water where the action of the Moon depresses it, and depresses it where the action of the Moon raises it: from the difference of their actions therefore arises the least, or, as they are called, Neap-tides. And, because the action U
of the Moon exceeds that of the Sun, high-water follows nearest the third lunar hour. At other times V
high-water arising from the lunar force would happen on the third lunar hour, and that which arises from the Sun's force on the third solar hour; but the forces being compounded, produce a tide which happens at some intermediate time, though always, on account of the greater force, nearest to the third lunar hour. Consequently, when the third solar W

hour precedes the third lunar hour, as is the caſe while the Moon is in the firſt and third quarters, high-water happens ſooner than the third lunar hour, and the contrary happens when the third lunar hour precedes the third ſolar hour, as in the
x ſecond and fourth quarters. It is to be noted, that no diſtinction is here made between the hour of the Sun or Moon's paſſing the meridian above the horizon, or beneath it; the effect being nearly the ſame with reſpect to the tides.

y The effects of the diſturbing forces of the Sun and Moon depend likewiſe upon their reſpective diſtances from the Earth. For theſe effects are
z greater at leſs diſtances. And therefore in winter, when the Earth is in its perihelium, the Sun being nearer, cauſes the ſpring-tides to be ſomewhat greater and the neap-tides ſomewhat leſs, than in
A the ſummer; and the Moon being each month in its perigeum, does then, in like circumſtances,
B cauſe greater tides than at other times; whence it happens, that if a great ſpring-tide happens when the Moon is in its perigeum, the next ſpring-tide will be leſs, becauſe the Moon will be then in its apogeum, or greateſt diſtance.

C The tides vary likewiſe in conſequence of the
D varying declinations of the Sun and Moon. For if either of theſe luminaries were ſuppoſed to be at the pole, it would neither accelerate nor retard the diurnal rotation of the water, but would occaſion a conſtant elevation at the poles, by diminiſhing the effect of gravity there, and a con-

ſtant depreſſion at the equator, from an oppoſite cauſe (219, c. 220, E). Therefore, no alternate riſe and fall of the water, or tide, would be produced. Conſequently, as the Sun and Moon decline towards the pole, they gradually loſe their effects, and the tides become leſs conſiderable. When the Sun is in the equator, and the Moon at the tropic, or its greateſt declination, the tides are leſs than when the Moon is at the equator, and the Sun in the tropic: becauſe in the firſt caſe the Sun's influence is the greateſt poſſible, and the Moon's leaſt; and in the latter the Moon's influence is the greateſt poſſible, and the Sun's leaſt: and as the tides depend more upon the Moon's influence than that of the Sun, they are greateſt when its action is greateſt. When the Sun and Moon are both in the equator, the ſpring-tides are the greateſt of any. However, becauſe the Earth is nearer the Sun in winter than in ſummer, the greateſt autumnal ſpring-tides are generally later than the equinox; and the greateſt vernal ſpring-tides are generally before the equinox.

When the Moon declines either to the northward or ſouthward of the equator, one of the greateſt elevations of the water follows the Moon, and deſcribes nearly the ſame parallel of latitude as the Moon, by the diurnal motion, apparently deſcribes; and the other greateſt elevation being diametrically oppoſite, muſt, of courſe, deſcribe a parallel of latitude at an equal diſtance on the other ſide of the equator. The greateſt elevation,

 which

which moves on the ſame ſide of the equator with a given place, will come nearer to it than the op-
K poſite elevation; and therefore when the Moon declines towards the ſame ſide of the equator, as that on which the given place is ſituated, the day-tides, or tides which happen while the Moon is above the horizon, will be greateſt, and the night-tides, or thoſe which happen while the Moon is beneath the horizon, will be leaſt. And the contrary happens when the Moon declines to the other
L ſide of the equator. Thus, the elevation at high-water is alternately greater and leſs; and the difference is greateſt when the Sun and Moon both deſcribe the ſame tropic, becauſe the oppoſite elevations then deſcribe the tropics, which are the fartheſt from each other of any two parallel circles they can poſſibly deſcribe. This difference is found to be about a foot at Plymouth, and fifteen inches at Briſtol.

M If the actions of the Sun and Moon were to ceaſe at once, the tides would not immediately ceaſe, but would continue for ſome time by the undulating motion of the water. This undulation would be greater, if the actions were to ceaſe at the time
N of a great tide than at the time of a leſs; and therefore leſs tides, which ſucceed greater, are more increaſed by it than greater tides which ſucceed leſs: conſequently the difference between the tides is rendered leſs than it would otherwiſe have been, and the greateſt ſpring and neap-tides do not happen preciſely at the conjunction or oppoſition, and quadratures, but two or three tides later.

If the greateſt acceleration and retardation of the diurnal motion (241, N) cannot ſubſiſt in the ſame ſea at the ſame time, the accumulation or defect muſt conſequently be leſs; that is to ſay, if one of the ſhores or coaſts of any ſea be leſs than ninety degrees to the eaſtward or weſtward of the other, and the eaſtern coaſt, for inſtance, be immediately under the Moon, the acceleration will, by cauſing the water to riſe, occaſion a defect or fall to the weſtward, becauſe the weſtern parts, being retarded, do not follow with a velocity ſufficient to ſupply what is carried to the eaſtward by the acceleration: and the greater this retardation the greater the defect or fall. But ſince by the ſuppoſition the weſtern ſhore is not 90 degrees diſtant, the retardation is not there ſo great as it would have been had the ſea been wider; and therefore the fall in that ſea is not ſo great. By a like argument it appears, that when the Moon is at the meridian of the weſtern coaſt the elevation is leſs, if the ſea be leſs than ninety degrees from eaſt to weſt. Hence in ſmall inland ſeas the tides are P inconſiderable; and for this cauſe, in the Atlantic ocean the tides do not riſe ſo high between the tropics as they do farther to the north or ſouth, the ſea being narrower between America and Africa in the lower than in the higher latitudes. From hence Q also follows the reaſon why the tides are ſo ſmall as they are found to be at the iſlands of St. Helena and Aſcenſion, which lie in the middle of that ſea; for, ſince the water cannot riſe on the ſhore but by falling at the other, it muſt continue at a mean height at theſe intermediate diſtant iſlands.

O

This theory of the tides is perfectly consentaneous to experience in the open and deep oceans; and in the lesser seas, as has been observed, the tides are very small. But, when those less seas have a free communication with the ocean, the tide flows into them in a kind of wave, which on its arrival at any place causes high-water. Thus it is high-water in the ocean to the westward of England and Ireland at the third lunar hour; after which it begins to subside. This subsidence must, of course, raise the water round about, whence a flood begins to enter the English channel at about the sixth hour, its course being retarded by the shallowness of the water. Another flood enters the German sea to the northward, near the Orkney islands, and proceeds to the southward. As these floods proceed on their respective courses, it is high-water successively at every place on the coasts at which they arrive, and when the wave has passed any place the water begins there to subside. For example; it is high-water at Plymouth about the sixth hour, at the Isle of Wight about the ninth hour, and at London-bridge about the fifteenth hour after the Moon has passed the meridian, and caused the tide in question. Therefore, when it is high-water at Plymouth the water out at sea has half subsided; when it is high-water at the Isle of Wight it is low water out at sea; and when it is high-water at London-bridge it is low water at the Isle of Wight, and a second flood or elevation has already come to its height out at sea.

There

There are situations where the tide may be carried to the same port by different passages or channels, and may pass quicker through one passage than another: in which case, the same tide, arriving at different times through the different passages, must occasion a variety of phenomena. Suppose two equal tides to arrive at the same port from different places; the one at the third, and the other at the ninth hour after the Moon has passed the meridian; the first tide therefore preceding the latter by six hours; and suppose the Moon to be at the equator: then, every six hours a tide will arrive, which, flowing in at the same time as the preceding equal tide ebbs out, will cause the water to continue at the same height, and thus it will neither rise nor fall during the whole day. But if the Moon decline from the equator, the tides in the ocean will be alternately greater and less, as has been observed; and therefore there will arrive at this port, alternately, two greater floods proceeding from the greater tide in the ocean, and two less floods proceeding from the lesser tide in the ocean. At the mean or intermediate time between the arrivals of the two greater tides, the water will then be highest; between a greater and a less tide it will be at a mean height; and lowest of all at the middle time between the arrivals of the two less tides. By these means, in the space of twenty-four hours, the sea will rise to its greatest, and fall to its least height but once, instead of twice, as in general it does in other places; and if the Moon decline towards the

same

ſame ſide of the equator, as that on which the port is ſituated, the two greater tides will arrive at the third and ninth hours, and the greateſt elevation will be at the ſixth hour, or at about the ſetting of the Moon: the leaſt elevation will conſequently happen between the two leaſt tides, at the eighteenth hour, or about the riſing of the Moon. And the ſame effects will take place when the Moon declines to the contrary ſide of the equator; but with this difference, that whereas high and low water happened then reſpectively at the ſetting and riſing of the Moon, they will in the preſent caſe happen reſpectively at the riſing and ſetting of the Moon.

w A remarkable inſtance of all theſe particulars is adduced by Dr. Halley, in the port of Batſha, in the kingdom of Tonquin, which lies in 20° 50′ north latitude. There, on the day on which the Moon paſſes the equator, the water ſtagnates; afterwards, on the Moon's declining to the northward, it begins to ebb and flow; not twice in the day, as in other ports, but once only; and high-water happens at the ſetting, and low-water at the riſing of the Moon. The tides increaſe with the Moon's declination for ſeven or eight days; after which, for the next ſeven days, they decreaſe by the ſame gradation as they before increaſed, till the Moon's arrival again at the equator, when they ceaſe, and upon its changing its declination are reverſed. For while its declination becomes ſoutherly, low-water happens at the ſetting, and high-water at the riſing

of the Moon; which continues till its declination again changes. Now it appears, that the tide must come to this port by two inlets or passages; one between the continent of China and the island Luconia, communicating with the Chinese ocean, and the other between the island of Borneo and the continent. But whether the tide arriving from the Indian ocean, after a course of twelve hours, and from the Chinese ocean after a course of six, and thence happening on the third and ninth lunar hours, be the cause of these appearances; or whether some other circumstances may not be concerned in producing the effect, must be determined by observations on the neighbouring coasts.

BOOK

BOOK II.

SECT. I.

Of Light.

CHAP. I.

CONCERNING THE MOTION OF LIGHT.

In the former part of this Treatiſe our attention x has been employed in conſidering ſuch effects as ariſe from the motions and mutual actions of bodies, of magnitude ſufficient to become the objects of our ſenſes. It may be eaſily ſeen, that phenomena of this kind are but few compared with the very great number of operations which ariſe from the actions of bodies too minute to be determined but by deductions or inferences made from their effects. We have contemplated the great outline of the univerſe, and its vaſtneſs cannot but excite the aſtoniſhment of creatures who are deſtined at preſent to occupy an exceedingly ſmall part of it. As we proceed to examine that ſmall part, we ſhall develope a ſcene of another kind, which, though expanded through a leſs portion of ſpace, is equally immenſe and unlimited with regard to the field it affords for admiration and perpetual diſcovery.

The ſucceſs of every enquiry depends in no ſmall y degree on the order employed in the ſeveral re-

ſearches to be made. It is obvious, that the probability of error is greater the more complicated the ſubject; whence the neceſſity of firſt examining the moſt ſimple, and thence proceeding to more compounded objects, is evident. This principle leads us, in our conſideration of the particular properties of various bodies, to attend firſt to thoſe of light.

z It is generally agreed, that light conſiſts of ſmall bodies or particles, projected with great velocity in all directions, from the luminous or radiant body. No ſolid objections have been made to this hypotheſis, which appears to be more ſimple than any other, and is perfectly conſiſtent with all the phenomena yet obſerved; and theſe are ſo many and ſo various as to leave very little doubt of its truth.

A The velocity of light was firſt determined by Monſ. Romer, from obſervations on Jupiter's Moons (140, w) and the meaſure deduced from his obſervations was afterwards confirmed and eſtabliſhed by the diſcovery of the aberration of the fixed ſtars. The principles on which this diſcovery is founded may be familiarly explained as follows *.

B Suppoſe a tube to be erected perpendicular to the horizon, at a time when it rains, the drops falling perpendicularly down, and ſuppoſe the diameter of the tube to be ſuch as to admit but one drop at a time: then it is plain, that if a

* The original account of this diſcovery may be ſeen in a paper by its inventor Dr. Bradley, inſerted in the Philoſophical Tranſactions for the year 1728, No. 406.

drop

drop of water enter the orifice of the tube it will fall to the bottom without touching its ſides. But if the tube, without altering its perpendicularity, be moved along in the direction of the horizon, any drop that enters will ſtrike againſt one of its ſides, and none will paſs clear through while the motion continues, unleſs the upper end of the tube be alſo inclined towards the part to which its motion is directed.

Thus, if A B (fig. 70) repreſent the horizon, C D the perpendicular tube, and G D the courſe of a drop of rain: then, if C D be moved towards A, while the drop is falling within the tube, it is evident that the inner ſurface of the tube, which is ſituated towards B, will be carried againſt the drop, and prevent its arriving at the bottom without touching. But if the inclined tube E C be moved with a ſimilar motion to that of the drop from E to D, in the time that the drop moves from C to D, the lower orifice of the tube and the drop will be found at D at the ſame inſtant; and the velocity of the drop will be expreſſed by C D, and that of the tube by E D.

The ſame reaſoning holds good, if inſtead of drops of rain we ſuppoſe particles of light, and a teleſcope inſtead of a tube. For to an obſerver, who through the tube C D views the vaſtly diſtant object G, if the motion of light be inſtantaneous, or infinitely ſwift, no finite motion of C D, its poſition being unaltered, can prevent its being viſible; ſince, by the ſuppoſition, the light which enters at

E will arrive at D before C D can have moved at all. But if light be propagated in time, and the observer be carried by a motion similar, as to acceleration, to that of light, the tube must be inclined to the ray in an angle, whose sine is to the sine of C E D, or the angle the tube makes with the line of the observer's motion, as the velocity of the observer is to the velocity of light. For in the triangle D C E, the sides D E and D C, which express these velocities, are as the sines of their opposite angles. Hence if the angle of the inclination of the tube to the ray of light, together with the velocity and direction of the observer's motion be known, the velocity of light may be determined.

E By this theory, which is established by a great number of observations on stars of different magnitudes and situations, it appears, that the small apparent motion the fixed stars have about their real places, which is called their Aberration, arises from the proportion which the velocity of the Earth's motion in her orbit bears to that of

F light. This proportion is found to be as 10210 to 1: from whence it follows, that light moves or is propagated as far as from the Sun to the Earth

G in 8′ 12″. And it likewise appears, that the velocity of light is uniform, and the same, whether original, as from the stars, or reflected, as from the satellites of Jupiter (140, w).

H The velocity of light being known, an estimate might be made of the magnitude of its particles, if we were in possession of good observations

tions of the effects of their momentum. For example, it is found, that a ball from a cannon at its first discharge flies with a velocity of about a mile in * eight seconds, and would therefore arrive at the Sun in thirty-two years, supposing it to move with unremitted velocity. And light, as was before observed, moves through that space in about eight minutes, which is two million times as fast. But the force with which bodies move are as their masses multiplied by their velocities (19, L); and conse- K quently if the particles of light were equal in mass to the two millionth part of a grain of sand, we should be no more able to endure their impulse than that of sand shot point blank from the mouth of a can non.

From several experiments † that well deserve L to be repeated, in which the Sun's rays were thrown, much condensed, upon a very light lever, suspended so as to be removeable horizontally, it seems probable that the momentum of light may be great enough to be rendered sensible by the velocity it communicates to bodies by its impulse.

The rarity of light is not less a matter of wonder M at the first consideration, than its velocity and the minuteness of its particles. For its rays cross each other in all directions without the least apparent disturbance. We can easily see through a small

* This varies according to the charge of powder and other circumstances.

† By Mr. Michell. See Priestley's Optics, p. 387.

hole, not exceeding the $\frac{1}{100}$ part of an inch, the objects, as the ſky, trees, houſes, &c. which occupy almoſt an entire hemiſphere. The light proceeding from all theſe objects muſt therefore paſs at the ſame time through the hole in a very great variety of directions before they arrive at the eye; yet it does not appear that viſion is in the leaſt diſturbed by that means.

N This is, however, explained without difficulty from a well known circumſtance relative to the organs of viſion. For the action of light produces an effect on the eye that is not inſtantaneous, but laſts a conſiderable time. Suppoſe the effect of light on the eye to continue without ſenſible diminution, after the light has ceaſed to act, for the $\frac{1}{300}$ part of a ſecond, and it will follow that a ſucceſſion of particles of light arriving at the eye, by equal intervals, to the number of three hundred in a ſecond, will be ſufficient to excite a conſtant and uniform ſenſation of the preſence of light. And ſince the velocity of light is ſuch that it paſſes through about one hundred and ſeventy thouſand geographical miles in a ſecond (140, w); this ſpace divided by 300, will give nearly 570 miles for the diſtance between each of the above-mentioned ſucceſſive particles. It is not therefore to be wondered, that the particles of light do not interrupt each other, when we attend to their extreme minuteneſs (257, K) and the very great diſtance at which they may follow each other without preventing the conſtancy of their effect.

That the effect of light on the eye remains for a o
time is evinced from ſeveral obſervations. We are continually ſhutting our eyes, or winking, and ſhould on thoſe occaſions loſe ſight of all the ſurrounding objects, if the effect of their light did not continue during the time the eye is ſhut. Again, if a ſtick, or any other object, be whirled round in a circle, there is a certain velocity beyond which the object will fill the whole circle. This experiment is vulgarly made with a lighted firebrand, and obviouſly ſhews that the impreſſion, made on the eye by the light of the firebrand, when in any given point of the circle, is ſufficiently laſting to remain till the firebrand has deſcribed the whole circle, and again renews its effect, which is by that means rendered continual and uniform. Every one muſt have been ſenſible of the ſtrong and laſting impreſſion the Sun's direct light makes upon the eye; and impreſſions of the ſame kind from other objects, though weaker, are much leſs ſo than is generally imagined.

With reſpect to the duration of the impreſſions P
of light, it has been obſerved, that the teeth of a cog-wheel in a clock * were ſtill viſible in ſucceſſion when the velocity of rotation brought 246 teeth through a given fixed point in a ſecond. In this caſe it is clear, that if the impreſſion made on the eye by the light reflected from any tooth, had laſted without ſenſible diminution for the 246th part of a ſecond, the teeth would have formed one

* Watſon on Time.

unbroken line, becauſe a new tooth would have continually arrived in the place of the anterior one before its image could have diſappeared. If a live coal be whirled round, it is obſerved*, that the luminous circle is complete when the rotation is performed in $\frac{8}{60}$ of a ſecond. In this inſtance we ſee that the impreſſion was much more durable than the former. Laſtly, if an obſerver ſitting in a room direct his ſight through a window, to any particular object out of doors, for about half a minute, and then ſhut his eyes, and cover them with his hands, he will ſtill continue to ſee the window, together with the outline of the terreſtrial objects bordering on the ſky. This appearance will remain for near a minute, though occaſionally vaniſhing and changing color, in a manner that brevity forbids our minutely deſcribing. From theſe facts we are authorized to conclude, that all impreſſions of light on the eye laſt a conſiderable time; that the brighteſt objects make the moſt laſting impreſſions; and that, if the object be very bright, or the eye weak, the impreſſion may remain for a time ſo ſtrong, as to mix with and confuſe the ſubſequent impreſſions made by other objects. In the laſt caſe the eye is ſaid to be dazzled by the light.

The ſpace through which light paſſes is called a medium; by which term reference is had to the quantity or denſity of the matter contained in the

* By M. D'Arcy. See Mem. de l'Acad. des Sciences, 1765; or Prieſtley's Optics, p. 634.

ſpace:

ſpace: thus, glaſs and air are mediums, but a vacuum, or ſpace abſolutely void, is a medium alſo.

When light paſſes through mediums, either ab- R
ſolutely void, or containing matter of an uniform denſity, and of the ſame kind, it always is found to proceed in ſtraight lines. Whence it follows, that the rarity of light increaſes as the ſquare of the diſtance from the radiant body. For the light which falls on the ſquare A B C D, (fig. 71) from the point R, at the diſtance R A, will be ſpread over a ſurface, a b c d, four times as large at twice the diſtance, or R a.

CHAP. II.

OPTICAL DEFINITIONS AND PRINCIPLES.

WHEN a ray of light paſſes out of one medium S
into another, and is bent out of its courſe at their common ſurface, this bending is called refraction.

When a ray of light proceeds to the common T
ſurface of two mediums, and inſtead of paſſing from the one into the other, is turned back into the firſt, this turning back is called reflection.

The angle of incidence is the acute angle which U
the line deſcribed by the ray of light makes with a line drawn perpendicular to the ſurface at the point of incidence.

The angle of reflection or refraction is the acute V
angle, which the line deſcribed by the ray of light

after reflection or refraction makes with the perpendicular to the surface at the point of incidence.

W Thus, if R S represent the common surface of two mediums, A C (fig. 72) a ray of light incident at C, and P Q a line intersecting the surface at right angles at C; then the angle A C P is the angle of incidence. If it be reflected at C, so as to return in the line C B, then the angle P C B is the angle of reflection: and if it be refracted at C, so as to proceed in the line C F, the angle Q C F is the angle of refraction.

X The angles of incidence, reflection, and refraction lie in one and the same plane.

Y The angle of reflection is equal to the angle of incidence.

Z If the refracted ray be returned directly back to the point of incidence, it shall be refracted into the line which was before described by the incident ray.

A The refractive powers of different mediums are nearly as their densities; that is to say, if a ray of light pass out of a rarer into a denser medium, it will in general be refracted towards the perpendicular, so that the angle of refraction will be less than the angle of incidence. And the contrary will happen if the ray pass out of a denser into a rarer medium (262, Z).

B The sine of the angle of incidence is either accurately or very nearly in a given ratio to the sine of the angle of refraction, in all obliquities of the incident ray.

All

All objects ſeen by reflection or refraction appear in that place or direction, from whence or in which the rays were laſt reflected or refracted to the eye.

Thus, if the ray A C (fig. 72) proceed from an D
object at A to C, and be thence reflected to the eye of the ſpectator at B, the object will be ſeen not at A but at T, in the direction of the reflected ray B C. And if the ray F C proceed from an object at F, and be refracted into the direction C A to the eye of a ſpectator at A, the object will be ſeen not at F but at N, in the direction of the refracted ray A C.

On this account it is that objects are ſeen in E
mirrors or looking-glaſſes, and that objects ſeen under water appear out of their true places. Let A B C D (fig. 73) repreſent a veſſel containing water, whoſe ſurface is F G, and let O repreſent an object at the bottom. Then, to an eye at E the object O will be ſeen at K, by means of the ray O L, which paſſing from a denſer to a rarer medium (262, A) is refracted from the perpendicular P Q into the direction L E. Or let A B C D (fig. 74) repreſent a veſſel ſo placed with reſpect to the candle E, that the ſhadow of the ſide A C may fall at D. Suppoſe it now filled with water, and the ſhadow will withdraw to d, the ray of light E A, inſtead of proceeding to D, being refracted to d. And there is no doubt but that an eye placed at d would ſee the candle at e in the direction of the refracted ray d A.

The foregoing principles are founded on experiment or obſervation, and the mathematical application of them to the rays of light which paſs through glaſſes, or are reflected from mirrors of various figures, conſtitutes that branch of the ſcience of optics which teaches the conſtruction of inſtruments. But it will be proper to avoid the explanation of theſe for the preſent, till an account has been given of the various reflexibility, refrangibility, and colours of light.

CHAP. III.

OF THE VARIOUS REFRANGIBILITY OF THE RAYS OF LIGHT.

G LIGHTS which differ in colour differ alſo in refrangibility, and the contrary.

H Let A B (fig. 75) repreſent a wedge or triangular priſm of glaſs, then the triangle A B C (fig. 76) may be conceived to be a ſection of the ſame, at right angles to its axis. Suppoſe J N to be a ray of light incident at N, and thence refracted to E, on the ſurface C B, where it is again refracted into the direction E M; ſuppoſe i n to be another ray parallel to the former, and conſequently incident at n, with the ſame angle. Now, if the ray i n have exactly the ſame capability or diſpoſition to be refracted by the priſm, as the ray J N, the angles of refraction will alſo be equal, and in

will,

will, when refracted into the directions n e and e m, ſtill continue parallel to the ray J N, which is refracted into N E and E M. But if it be more refrangible it will be refracted into directions, as n f and f g, verging more towards the baſe A C, or, if leſs refrangible, it will be refracted into directions, as n h and h k, that verge leſs towards the baſe A C. Whence it appears, that if a pencil or collection of rays fall parallel to each other on one of the ſides of a priſm, and do not proceed parallel to each other on their emergence, it muſt be becauſe ſome of the rays are more refrangible than others.

Let the ſpace contained between E G and M R (fig. 77) repreſent a darkened chamber, of which thoſe lines repreſent the ſides. Let J N repreſent a pencil of light from the Sun, paſſing through the hole F, and incident on the ſide B C of the priſm A B C. It is plain, that if the priſm were not interpoſed, the pencil J N would proceed to S, and there illuminate a ſmall circular ſpot on the wall; and from the preceding explanation it is evident, that if all the rays of light be equally refracted by the priſm, the whole pencil, being equally turned out of its courſe, will ſuffer no alteration with reſpect to the paralleliſm of its rays, and conſequently will, after refraction, proceed to Q, and there illuminate a ſpot ſimilar to that which would have appeared at S. But if the pencil be compoſed of rays not alike refrangible, the moſt refrangible rays will be thrown farther from S, and the

the leaſt refrangible, being leſs deflected out of their courſe, muſt fall on a part of the wall nearer to s, while thoſe which are refrangible in the intermediate degrees will fall at interpoſed diſtances

K between them. The actual experiment determines, that the Sun's light is compoſed of rays, whoſe refrangibilities are not all the ſame; for after emerging from the priſm, inſtead of illuminating a circular ſpace, they are ſpread into a long ſpectrum, bounded by right-lined ſides and circular ends, and whoſe length is at right angles to the direction of the axis of the priſm.

L This oblong ſpectrum is variouſly coloured. The lower part, which conſiſts of the leaſt refrangible rays, is of a lively red, which, higher up by inſenſible gradations becomes an orange; the orange in like manner is ſucceeded by a yellow; the yellow by a green; the green by a blue; after which follows a deep blue or Indigo; and laſtly, a faint violet. With a priſm, whoſe refracting angle was $63\frac{1}{2}$ degrees, ſo placed, that the angle of incidence on the firſt ſurface was equal to the angle of refraction at the emergence or ſecond ſurface, the ſpectrum, received on a wall at the diſtance of $18\frac{1}{2}$ feet, was 10 inches, or $10\frac{1}{8}$ in length. Its breadth is in all caſes equal to that of the circle, which would have been formed at that diſtance by the admitted beam or pencil of light, if the priſm had not been interpoſed.

M The ſpaces occupied by the ſeveral colours of the ſpectrum anſwer to the ſubdiviſions of a muſical

ſical chord: thus, if A G M F (fig. 79) repreſent the ſpectrum, and the lines F M, b a, d g, z y, &c. mark the confines of the colours; the ſpace M a b F being occupied by the red, a g d b by the orange, g y z d by the yellow, and ſo forth, in the order above-mentioned, and G M be prolonged to x, ſo that M x may be equal to G M, the lines G x, l x, i x, e x, y x, g x, a x, M x, will be in proportion to each other as the numbers 1, $\frac{8}{9}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{9}{16}$, $\frac{1}{2}$, and therefore expreſs the chords of the key, tone, leſs third, fourth, fifth, greater ſixth, greater ſeventh, and octave. Or, to render it more familiar to thoſe whoſe knowledge of muſic is merely practical, let A B repreſent the ſtring of a violin or guittar, and a fret or ſmall bridge be fixed on the finger-board at the middle diſtance a, between A and B; and likewiſe frets at g, f, e, d, c, b, ſo that the diſtances a g, g f, f e, e d, d c, c b, and b A, may be in proportion to the ſpaces occupied reſpectively by the red, orange, yellow, green, blue, indigo, and violet, in the ſpectrum; then if the open ſtring ſound A, the regular aſcent of the ſtopt notes will be in the minor third from that key; the notes being A, B, C, D, E, F ſharp, G ſharp, and A octave.

Thoſe rays which have the ſame degree of re- N
frangibility will, after refraction, fall within a circle equal to that which would have been illuminated by the light if ſuffered to proceed to S (264, H): and therefore the ſpectrum may be conceived to be compoſed of an indefinitely great number

number of ſuch equal circles, whoſe centers are all on the ſame line. For example; if A B D C (fig. 78) repreſent the ſpectrum, the circle A B being formed by the red, or leaſt refrangible rays, and the circle D C by the violet, or moſt refrangible rays, then the rays, whoſe refrangiblity is intermediate, will form an innumerable ſeries of circles, and fill up the whole ſpace, ſo that A C and B D will appear as right
O lines. Now, it is obſervable, that though the light in the ſpectrum thus ſeparated into its original rays is much leſs compounded than before, yet it is ſtill compounded in no ſmall degree by the interference of the circles with each other, particularly at the line E F, equidiſtant between A C and B D; and that at the lines A C and B D, where the circles do not interfere at all, the light is perfectly homogeneous
P or uncompounded. But becauſe the colours in the ſpectrum contiguous on either ſide of any given colour do by mixture compound a colour that differs inſenſibly from the original intermediate colour itſelf, a right line drawn perpendicularly acroſs the ſpectrum will be found in the ſame colour
Q throughout. For moſt experiments in which uncompounded light is required, that of the ſpectrum will be found ſufficiently ſo, but in caſes where a greater nicety is demanded, the common diameter of the circles, or breadth of the ſpectrum, may be diminiſhed by making the hole at F ſmaller, or the figure of the hole may be altered..

R It is evident from what has been already ſaid, that this phenomenon ariſes from the nature of light itſelf, ſome

ſome of the rays of which are more refrangible than others. And as an additional confirmation it is s
obſerved, that if the ſpectrum be received on a board which is perforated, ſo as to let paſs one ray of light, or colour, that ray will not be changed by any refraction it may be afterwards made to ſuffer, but continues the ſame both in colour and refrangibility. And if the colours of the ſpectrum T
be by any reflection or refraction made to unite again, they will again form the compounded colour of whiteneſs.

The quantity of the diſperſion of the rays of u
light, which at equal diſtances from the priſm is nearly expreſſed by the length of the ſpectrum, does not follow the quantity of the refraction of the mean ray, except in mediums of the ſame kind. Thus, if two priſms of different kinds of glaſs re- v
fract the ſolar ray equally out of its firſt direction, the ſpectrum of colours formed by the one will be much longer than that formed by the other; and it is found, that in equal angles of mean refraction, w
glaſs, in the compoſition of which much lead enters, diſperſes the light into its component colours much more than glaſs which abounds with alkaline ſalts*.

If by means of two priſms, a ſmall piece of pa- x
per be illuminated, the one half with red, and the other half with violet light, and an obſerver view the ſame through another priſm, the paper will, by the different refrangibility of the rays, appear di-

* Acta (or perhaps Miſcellanea) Berolinenſia, for 1766; quoted by Prieſtley in his Optics, p. 474.

vided

vided into two. For the violet half being ſeen by a more refrangible light, will appear to be carried farther from its true place than the red, and will
Y therefore ſeem to be ſeparated from it. The ſame is likewiſe true of colours which ariſe from the ſeparation of light which is made by bodies on which it falls, and which we are apt to call natural colours; for if a paper be painted, the one half with a lively red, and the other half with an indigo, and it be placed in the Sun's light, it will in the ſame manner appear divided, if viewed through a priſm.

CHAP. IV.

OF THE VARIOUS REFLEXIBILITY OF THE RAYS OF LIGHT.

Z The Sun's light conſiſts of rays which differ in reflexibility, and thoſe rays which are more refran-
A gible are alſo more reflexible than others. Let A B C (fig. 80) repreſent a priſm, whoſe angle B is a right angle, and the two angles A and C equal to each other. Suppoſe J N to be a beam of light which paſſes through the ſurface B C, and is incident on A C at N. It will then emerge in the direction N G, ſo that the ſine of the angle of refraction G N W may be in a certain ratio, (262, B) to the ſine of the angle of incidence B N Z, which in glaſs is as 3 to 2, nearly. Now, when the angle of incidence at N is ſuch, that the ſine of the angle of refraction is equal to the radius, the angle of refraction

fraction becoming a right angle, the ray cannot emerge, but will be totally reflected or turned back into the glaſs. This happens in glaſs when the angle of incidence is about 41 degrees.

That the component rays of light are not at B all equally diſpoſed to be reflected, is proved by turning the priſm ſlowly on its axis, till the light begins to be reflected; for it then appears, that the more refrangible rays are reflected ſooner, or at leſs angles of incidence than thoſe which are leſs refrangible. Let N M repreſent the reflected beam, and C ſuppoſe the priſm V X Y placed ſo as to receive and ſeparate it into its component colours by refraction: then the light which firſt begins to be reflected, conſiſting almoſt intirely of violet rays, will by the ſecond priſm be refracted ſo as to fall at p, and paint a violet colour. As the firſt priſm continues to be turned on its axis, the light is more and more copiouſly reflected, and the colours between p and t appear in ſucceſſion according to their order in refrangibility; violet, indigo, blue, green, yellow, orange, and laſtly red, at which time the reflection becomes total: the colours formed by refraction at H G diſappearing as thoſe at p t appear.

White light being proved to conſiſt of rays D which differ in refrangibility, reflexibility, and the power of exciting the idea of colour; it is clear that nothing more is neceſſary to account for the colours of bodies than to ſuppoſe each body endued with a power or aptitude to reflect the rays of one particular colour, and to imbibe the reſt.

But

But the truth of this does not reſt on mere ſuppo-
E ſition. Bodies expoſed to the uncompounded light of the ſpectrum, are ever found to be of the colour of the light in which they are placed, with this only difference, that they appear much more lively in that colour, which is the ſame with that which they
F exhibit in the day light. And from hence it appears, that the colours of bodies cannot be ſo homogeneous and full as thoſe of the ſpectrum; for ſince they reflect all colours in ſome degree as well as the principal or predominant one, that principal colour muſt be much diluted and weak-
G ened by the mixture. It may likewiſe from hence be inferred, that as the uncompounded colours are not changeable by refraction, ſo neither are they changeable by reflection.

H Language being invented chiefly for the expreſſion of ordinary events that do not require any great preciſion, it very frequently happens, that the ſame word is uſed to denote very different things. It is proper to be remarked, that the word colour is thus uſed. If the word be uſed to denote the ſenſation or idea excited in the mind, it is ſufficiently obvious, that it cannot be ſcientifically uſed to denote that attribute by which bodies are able, by reflecting the rays of light, to produce the ſenſation. And ſtill leſs ought it to be uſed to imply that quality the various kinds of light poſſeſs, of producing the ſenſation, when ſeparated from each other, either by reflection from bodies or otherwiſe. It may, however, be allowed to uſe

the terms coloured rays, or coloured bodies, though the ſenſation of colour, the ſpecific properties of the rays, and of the reflecting bodies, are undoubtedly things very different from each other. So different, indeed, that this remark might with juſtice be ſuppoſed unneceſſary, if experience had not ſhewn, that among the pretenders to philoſophical knowledge ſome have been ſound capable of miſtaking in this very particular *.

CHAP. V.

CONCERNING THE RAINBOW.

THE inſtance of the ſeparation of the primary I
colours of light which ſeems moſt remarkable, is
that of the rainbow. It is formed in general by the K
reflection of the rays of the Sun's light from the
drops of falling rain, though frequently it appears
among the waves of the ſea, whoſe heads or tops
are blown by the wind into ſmall drops, and is ſometimes
ſeen on the ground when the Sun ſhines on a
very thick dew. Caſcades and fountains, whoſe
waters are in their fall divided into drops, exhibit
rainbows to a ſpectator, if properly ſituated during
the time of the Sun's ſhining; and water blown violently
out of the mouth of an obſerver, whoſe back
is turned towards the Sun, never fails to produce
the ſame phenomenon. This appearance is alſo L

* The oppoſers of Newton's diſcoveries on light and colours have falſely affirmed, that he taught that the rays of light were coloured.

ſeen by moonlight, though ſeldom vivid enough to render the colours diſtinguiſhable; and the artificial rainbow may be produced even by candle-light on the water which is ejected by a ſmall fountain or jet d'eau. All theſe are of the ſame nature, and dependant on the ſame cauſes, an idea of which may be formed by the following conſiderations.

M Let the circle W Q G B (fig. 81) repreſent a globe or drop of water upon which a beam of parallel light falls, of which let T B repreſent a ray falling perpendicularly at B, and which by conſequence (262, Y, B) either paſſes through without refraction, or is reflected directly back from Q. Suppoſe another ray I K, incident at K, at a diſtance from B, and it will be refracted according to a certain ratio (262, B) of the ſines of incidence and refraction to each other, which in rain water is as 529 to 396, to a point L, whence it will be in part tranſmitted in the direction L Z, and in part reflected to M, where it will again in part be reflected, and in part tranſmitted in the direction M P, being inclined to the line deſcribed by the incident ray in the angle I O P. Another ray A N, ſtill farther from B, and conſequently incident under a greater angle, will be refracted to a point F, yet farther from Q, whence it will be in part reflected to G, from which place it will in part emerge, forming an angle A X R with the incident A N, greater than that which was formed between the ray M P and its incident ray. And

And thus, while the angle of incidence or diſtance of the point of incidence from B increaſes, the diſtance between the point of reflection and Q, and the angle formed between the incident and emergent reflected rays will alſo increaſe; that is to ſay, as far as it depends on the diſtance from B: but as the refraction of the ray tends to carry the point of reflection towards Q, and to diminiſh the angle formed between the incident and emergent reflected ray, and that the more the greater the diſtance of the point of incidence from B, there will be a certain point of incidence between B and W, with which the greateſt poſſible diſtance between the point of reflection and Q, and the greateſt poſſible angle between the incident and emergent reflected ray will correſpond. So that a ray incident nearer to B ſhall, at its emergence after reflection, form a leſs angle with the incident, by reaſon of its more direct reflection from a point nearer to Q; and a ray incident nearer to W, ſhall at its emergence form a leſs angle with the incident, by reaſon of the greater quantity of the angles of refraction at its incidence and emergence. The rays which fall for a conſiderable ſpace in the vicinity of that point of incidence with which the greateſt angle of emergence correſponds, will, after emerging, form an angle with the incident rays differing inſenſibly from that greateſt angle, and conſequently will proceed nearly parallel to each other; and thoſe rays which fall at a diſtance from that point will emerge at various angles, and conſequently will diverge. Now, to a ſpectator, whoſe back is turned

towards the radiant body, and whoſe eye is at a conſiderable diſtance from the globe or drop, the divergent light will be ſcarcely, if at all perceptible; but if the globe be ſo ſituated, that thoſe rays which emerge parallel to each other, or at the greateſt poſſible angle with the incident, may arrive at the eye of the ſpectator, he will, by means of thoſe rays, behold it nearly with the ſame ſplendor at any diſtance.

N In like manner, thoſe rays which fall parallel on a globe, and are emitted after two reflections, ſuppoſe at the points F and G, will emerge, at H, parallel to each other, when the angle they make with the incident, A N, is the leaſt poſſible; and the globe muſt be ſeen very reſplendent, when its poſition is ſuch, that thoſe parallel rays fall on the eye of the ſpectator.

O The quantities of theſe angles are determined by calculation, the proportion of the ſines of incidence and refraction to each other being known. And this proportion being different (264, G) in rays which produce different colours, the angles muſt vary in each. Thus it is found, that the greateſt angle in rain-water for the leaſt refrangible, or red rays, emitted parallel after one reflection is 42° 2′, and for the moſt refrangible or violet rays emitted parallel after one reflection 40° 17′; likewiſe, after two reflections the leaſt refrangible or red rays will be emitted nearly parallel under an angle of 50° 57′, and the moſt refrangible or violet under an angle of 54° 7′; and the intermediate colours

will

will be emitted nearly parallel at intermediate angles.

Suppose now, that O (fig. 82) is the spectator's eye, and O P a line drawn parallel to the Sun's rays, and let P O E, P O F, P O G, P O H, be angles of 40° 17′, 42° 2′, 50°, 57′, and 54° 7′ respectively, and these angles turned about their common side O P, will, with their other sides O E, O F; O G, O H describe the verges of two rainbows as in the figure. For, if E, F, G, H be drops placed any where in the conical superficies described by O E, O F, O G, O H, and be illuminated by the Sun's rays S E, S F, S G, S H; the angle S E O being equal to the angle P O E, or 40° 17′, will be the greatest angle in which the most refrangible rays can, after one reflection, be refracted to the eye, and therefore all the drops in the line O E must send the most refrangible rays most copiously to the eye, and thereby strike the sense with the deepest violet colour in that region. And in like manner the angle S F O being equal to the angle P O F, or 42° 2′, will be the greatest in which the least refrangible rays after one reflection can emerge out of the drops, and therefore those rays must come most copiously to the eye from the drops in the line O F, and strike the sense with the deepest red colour in that region. And, by the same argument, the rays which have the intermediate degrees of refrangibility will come most copiously from drops between E and F, and strike the senses with the intermediate colours in the order which their degrees of refrangibility require;

 that

that is, in the progreſs from E to F, or from the inſide of the bow to the outſide, in this order, violet, indigo, blue, green, yellow, orange, red. But the violet, by mixture of the white light of the clouds, will appear faint, and inclined to purple.

Q Again, the angle S G O being equal to the angle P O G, or 50° 57′, will be the leaſt angle in which the leaſt refrangible rays can, after two reflections, emerge out of the drops, and therefore the leaſt refrangible rays muſt come moſt copiouſly to the eye from the drops in the line O G, and ſtrike the ſenſe with the deepeſt red in that region. And the angle S H O being equal to the angle P O H, or 54° 7′, will be the leaſt angle in which the moſt refrangible rays, after two reflections, can emerge out of the drops, and therefore thoſe rays muſt come moſt copiouſly to the eye from the drops in the line O H, and ſtrike the ſenſe with the deepeſt violet in that region. And, by the ſame argument, the drops in the regions between G and H will ſtrike the ſenſe with the intermediate colours in the order which their degrees of refrangibility require; that is, in the progreſs from G to H, or from the inſide of the bow to the outſide in this order, red, orange, yellow, green, blue, indigo, and violet. And ſince the four lines O E, O F, O G, O H may be ſituated any where in the above-mentioned conical ſuperficies, what is ſaid of the drops and colours in theſe lines is to be underſtood of the drops and colours every where in thoſe ſuperficies.

R Thus there will be made two bows of colours, an

an interior and ſtronger, by one reflection in the drops, and an exterior and fainter by two; for the light becomes fainter by every reflection; and their colours will lie in a contrary order to each other, the red of both bows bordering upon the ſpace G F, which is between the bows. The breadth of the interior bow, E O F, meaſured croſs the colours, will be 1° 45′, and the breadth of the exterior, G O H, will be 3° 10′, and the diſtance between them G O F, will be 8° 55′, the greateſt ſemidiameter of the innermoſt, that is, the angle P O F, being 42° 2′, and the leaſt ſemidiameter of the outermoſt P O G being 50° 57′. Theſe are the meaſures of the bows, as they would be, were the Sun but a point; for, by the breadth of its body, the breadth of the bows will be increaſed, and their diſtance diminiſhed by half a degree, and ſo the breadth of the interior iris will be 2° 15′, that of the exterior 3° 40′, their diſtance 8° 25′; the greateſt ſemidiameter of the interior bow 42° 17′, and the leaſt of the exterior 50° 42′. And ſuch are the dimenſions of the bows in the heavens found to be very nearly, when their colours appear ſtrong and perfect.

The light which comes through drops of rain by two refractions without any reflection ought to appear ſtrongeſt at the diſtance of about 26 degrees from the Sun, and to decay gradually both ways as the diſtance from the Sun increaſes and decreaſes. And the ſame is to be underſtood of light tranſmitted through ſpherical hail-ſtones. And if the

hail be a little flatted, as it often is, the light tranſmitted may grow ſo ſtrong at a little leſs diſtance than that of 26 degrees, as to form a halo about the Sun and Moon; which halo, as often as the ſtones are duly figured, may be coloured, and then it muſt be red within, by the leaſt refrangible rays, and blue without, by the moſt refrangible ones.

T The light which paſſes through a drop of rain after two refractions, and three or more reflections, is ſcarcely ſtrong enough to cauſe a ſenſible bow.

CHAP. VI.

OF THE SEPARATION OF THE ORIGINAL RAYS OF LIGHT BY REFLECTION OR TRANSMISSION, THAT DEPENDS ON THE THICKNESS OF THE MEDIUM UPON WHICH THEY ARE INCIDENT.

U THE original or component rays of light are ſeparable from each other, not only by refraction, or by varying the angle of incidence on a reflecting ſurface, but are likewiſe at like incidences more or leſs reflexible, according to the thickneſs or diſtance between the two ſurfaces of the medium on which
V they fall. They are alſo liable to be turned out of their direct courſe by approaching within a certain diſtance from a body, by which means a ſeparation enſues, the rays being more or leſs deflected as they differ

differ in colour. Of theſe circumſtances it will be proper to give ſome account.

If a glaſs or lens, whoſe ſurface is convex, or a w portion of a ſphere, be laid upon another plain glaſs, it is evident that it will reſt or touch at one particular point only; and therefore, that at all other places between the adjacent ſurfaces will be interpoſed a thin plate of air, the thickneſs of which will increaſe in a certain ratio, according to the diſtance from the point of contact; that is to ſay, in arcs whoſe verſed ſines are very ſmall, as the diameter of the ſphere is to the ſine of the arc, ſo is that ſine to the verſed ſine or thickneſs of the air at the diſtance meaſured by the ſine.

Light incident upon ſuch a plate of air is diſ- x poſed to be tranſmitted or reflected according to its thickneſs: thus, at the center of contact, the light is tranſmitted, and a black circular ſpot appears; this ſpot is environed by a circle, the colours of which, reckoning from the internal part, are blue, white, yellow, red; then follows another circular ſeries, viz. violet, blue, green, yellow, red; then purple, blue, green, yellow, red; green, red; greeniſh blue, red; greeniſh blue, pale red; greeniſh blue, reddiſh white.

Theſe are the colours which appear by reflec- y tion: by the tranſmitted light the following ſeries are ſeen. At the centre white, then yellowiſh red, black; violet, blue, white, yellow, red; violet, blue, green, yellow, red, &c. ſo that the tranſmitted light at any thickneſs, inſtead of white,

appears

appears of the compounded colour which it ought to have after the ſubtraction of ſome of the conſtituent colours by reflection; after which ſeries the colours become too faint and dilute to be diſcerned.

z It is obſervable, that the glaſſes will not come into contact without a conſiderable degree of preſſure.

A By admeaſurement it appears, that the rays of any particular colour are diſpoſed to be reflected when the thickneſſes of the plate of air are as the numbers 1. 3. 5. 9. 11. &c. and that the ſame rays are diſpoſed to be tranſmitted at the intermediate thickneſſes which are as the numbers o. 2. 4. 6. 8. 10. &c.

B The places of reflection or tranſmiſſion of the ſeveral colours in a ſeries are ſo near each other, that the colours dilute each other by mixture, whence the number of ſeries in the open day-light ſeldom exceeds ſeven or eight: but if the ſyſtem be viewed through a priſm, by which means the rings of various colours are ſeparated according to their refrangibilities, they may be ſeen on that ſide towards which the refraction is made, ſo numerous,

C that it is impoſſible to count them. Or, if in a dark chamber the Sun's light be ſeparated into its original rays by a priſm, and a ray of one uncompounded colour be received upon the two glaſſes heretofore deſcribed, the number of circles will become very numerous, and both the reflected and tranſmitted light will remain of the ſame colour as the original incident ray. In this experiment it alſo is ſeen, that in any ſeries, the circles formed by

by the less refrangible rays exceed in magnitude those which are formed by the more refrangible rays, and consequently that in any series the more D refrangible rays are reflected at less thicknesses than those which are less refrangible.

If the light be incident obliquely, the rings of E colours dilate and enlarge themselves; whence it follows, that the thickness required to reflect the colours of any series is different in different obliquities.

Water, applied to the edges of the glasses, is F attracted between them, and filling all the intercedent space, becomes a thin plate of the same dimensions as that which before was constituted of air. In this case the rings become much fainter, but vary not in their species, and are contracted in diameter nearly in the proportion of 7 to 8: consequently the intervals of the glasses at like circles caused by these two mediums, water and air, are as about 3 to 4: that is, nearly as the sines which measure the angles of incidence and refraction, made at a common surface between them. And hence it may be suspected, that if any other G medium, more or less dense than water, be compressed between the two glasses, their intervals at the rings caused thereby will be to the intervals at which similar rings are caused by the interjacent air, as the sine which measures the refraction made out of air, into that medium is to the sine of the incidence on the common surface.

These are some of the phenomena of light inci- H dent

dent on mediums which are environed by mediums of greater denſity, as air or water compreſſed or included between plates of glaſs. The ſame appearances follow, though with ſome little variation, when the colorific medium is denſer than that in which it is incloſed.

I It is well known that bubbles blown in ſoap-water exhibit a great variety of colours; but as theſe colours are commonly too much agitated by the external air to admit of any certain obſervation, it is neceſſary that the bubble be covered with a clear glaſs; in which ſituation the following appearances enſue: the colours emerge from the vertex or top of the bubble, and as it grows thinner by the ſubſidence of the water, they dilate into circles or rings parallel to the horizon, which ſlowly deſcend and vaniſh ſucceſſively at the bottom. This emergence continues till the water at the vertex becomes too thin to reflect the light, at which time a circular ſpot of an intenſe blackneſs appears at the top, which ſlowly dilates ſometimes to three quarters of an inch in breadth before the bubble breaks. Reckoning from the black central ſpot, the reflected colours are the ſame in ſucceſſion and quality as thoſe produced by the aforementioned plate of air, and the appearance of the bubble, if viewed by tranſmitted light, is alſo ſimilar to that of the plate of air in like circumſtances.

K If the colours be viewed with different obliquities,

ties, their place is changed, but not near so much as in the plate of air.

The end of a small glass tube or pipe being melted, by turning the flame of a candle or lamp upon it, by means of a blow-pipe, may be blown into a bubble of an extreme thickness. Such a bubble will exhibit colours of the same kind as the foregoing, but much more brisk and lively. From which, and the premised observations, it is concluded that a denser medium inclosed by one that is rarer exhibits more lively colours than those which are produced by a rarer medium included in one that is more dense. It is also observable, that the colours produced by reflection from, or transmission through, dense substances, are less subject to vary by change of the obliquity of the incident light than they are in substances that are more rare.

L (margin) M (margin) N (margin)

By wetting very thin plates of Moscovy glass, whose thinness occasion the like colours to appear, the colours become more faint and languid, especially if wetted on the surface opposite to the eye; but no variation of their species is produced: so that the thickness of any plate requisite to produce any colour, seems to depend only on the density of the plate, and not on that of the ambient medium: and hence, if the suspicion formerly urged be true (283, o), may be known the thickness which thin plates of any transparent substance have at the place where a given colour in any series is produced. For, O

As

As the fine of the angle of incidence at the common furface
Is to the fine of the angle of refraction out of the given medium into air,
So is the thicknefs of a plate of air which exhibits the given colour
To the thicknefs of the given plate.

P As lenfes ground to a long radius are neceffary to be ufed in thefe experiments, and fuch are not very common, it may be an acceptable piece of information for th learner to know, that their place may be well fupplied by two pieces of plate-glafs, or even common glafs. If thefe be previoufly wiped, and then rubbed together, they will foon adhere with a confiderable degree of force, and exhibit various ranges of colours, much broader than thofe obtained by lenfes. One of the moft remarkable circumftances attending this method of making the experiment is the facility with which the colours may be removed, or even made to difappear by heats too low to feparate the glaffes. It feems moft probable, that the operation of heat confifts in augmenting the diftance between the furfaces. A touch of the finger immediately caufes the irregular rings of colours to contract towards their center in the part touched.

CHAP.

CHAP. VII.

GENERAL INFERENCES RESPECTING THE DISPOSITION TO BE REFLECTED OR TRANSMITTED, INTO WHICH THE RAYS OF LIGHT ARE PUT, BY THE ACTION THAT DEPENDS ON THE THICKNESS OF THE MEDIUM UPON WHICH THEY ARE INCIDENT.

THE experiments or obſervations in the laſt chapter being maturely weighed and conſidered, indicate the following theorem or general propoſition; namely, Q

Every ray of light in its paſſage through any refracting ſurface is put into a certain tranſient conſtitution or ſtate, which in the progreſs of the ray returns at equal intervals, and diſpoſes the ray, at every return, to be eaſily tranſmitted through the next refracting ſurface, and, between the returns, to be eaſily reflected by it. R

For, by thoſe obſervations it appears, that one and the ſame ſort of rays, at equal angles of incidence on any thin tranſparent plate, is alternately reflected and tranſmitted for many ſucceſſions; accordingly, as the thickneſs of the plate increaſes in arithmetical progreſſion of the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, &c. ſo that if the firſt reflection, or that which makes the firſt or innermoſt ring of colours, be made at the thickneſs 1, the rays ſhall be S

be tranſmitted at the thickneſſes 0, 2, 4, 6, 8, 10, 12, &c. and thereby make the central ſpot and rings of light which appear by tranſmiſſion, and be reflected at the thickneſſes 1, 3, 5, 7, 9, 11, &c. and thereby make the rings which appear by reflection. And this alternate reflection and tranſmiſſion continues for a great number of viciſſitudes, and by other obſervations, which for the ſake of brevity are omitted, for many thouſands, being propagated from one ſurface of a glaſs-plate to the other, though the thickneſs of the plate be a quarter of an inch or above: ſo that this alternation ſeems to be propagated from every refracting ſurface to all diſtances without end or limitation. And becauſe the ray is diſpoſed to reflection at the thickneſſes 1, 3, 5, 7, &c. and to tranſmiſſion at the thickneſſes 0, 2, 4, 6, 8, &c. for its tranſmiſſion through the firſt ſurface is at the diſtance 0, and it is tranſmitted through both together, if their diſtance be infinitely little, or much leſs than 1, the diſpoſition to be tranſmitted at the diſtances 2, 4, 6, 8, &c. is to be accounted a return of the ſame diſpoſition which the ray firſt had at the diſtance 0, that is, at its tranſmiſſion through the firſt refracting ſurface.

T This alternate reflection and tranſmiſſion depends on both the ſurfaces of very thin plate, becauſe it depends on their diſtance. For if either ſurface of a thin plate of Muſcovy-glaſs be wetted, the colours grow faint (285, 0): it muſt therefore depend upon both.

It

It is therefore performed at the second surface; u
for if it were performed at the first, before the rays
arrive at the second, it would not depend on the
second.

It is also influenced by some action or disposi- v
tion, propagated from the first to the second, be-
cause otherwise at the second it would not depend
upon the first. And this action or disposition, in
its propagation, intermits and returns by equal in-
tervals, because in all its progress it inclines the
ray at one distance from the first surface to be re-
flected by the second, at another to be transmitted
by it, and that, by equal intervals, for innumerable
vicissitudes.

The returns of the disposition of any ray to be w
reflected are termed its fits of easy reflection, and
those of its disposition to be transmitted its fits of
easy transmission; and the space it passes through
between every return, and the next return, the in-
terval of its fits.

Thus, let O D F E (fig. 83) represent a transpa- x
rent medium, suppose water, upon which the ray
A B is incident at a point in the upper surface
O, O. Draw the line I, I, and let the interval be-
tween it and O, O, be every where equal to the dis-
tance between the two surfaces of the plate of water,
described in the last chapter (283, F), when the
first ring of colour is reflected. Then if the in-
ferior surface of the medium were at I, I, the ray
would be reflected upon the same principle as the
ring of colour, and therefore at I, I it is in a fit

of eafy reflection. Draw the parallel 2, 2 at the fame diftance from 1, 1, and the diftance between o, o, and 2, 2 will be that thicknefs at which in the before-mentioned plate the firft ring of colour is tranfmitted: the ray would therefore be tranfmitted if the inferior furface were at 2, 2, and confequently it is there in a fit of eafy tranfmiffion. At 3, 3 it is again in a fit of eafy reflection, and by applying the fame argument to the equidiftant lines 4, 4; 5, 5; 6, 6; 7, 7; 8, 8; it will appear that the ray will be alternately difpofed to tranfmiffion and reflection; and if the laft parallel or the inferior furface be diftant from the fuperior furface o, o, by an even number of intervals, the ray will arrive there in a fit of eafy tranfmiffion and emerge; but if the number be odd, it will arrive in a fit of eafy reflection, and return back into the medium. The diftance between the lines o, o and 2, 2; 2, 2 and 4, 4, &c. are the intervals of the fits of eafy tranfmiffion, and the diftances between 1, 1 and 3, 3; 3, 3 and 5, 5, &c. are the intervals of the fits of eafy reflection.

What kind of action or difpofition this may be, whether it confift in a circulating or a vibrating motion of the ray or of the medium, or fomething elfe, experiments are wanting to determine. But the facts are not the lefs true on account of our ignorance of the mode of their origin. That truly great man, to whofe penetration and induftry we are indebted for almoft all the knowledge we have of the phyfical properties of light, has, with

great

great modesty, proposed an hypothesis for the solution of these appearances. It is not without its difficulties, and must therefore be received with the same caution as it was proposed, till experiment shall either confirm it, or substitute another theory in its place.

Sir Isaac Newton's Hypothesis. It may be supposed, that as stones by falling into water put the water into an undulating motion, and all bodies by percussion excite vibrations in the air; so the rays of light, by impinging on any refracting or reflecting surface, excite vibrations in the refracting or reflecting medium or substance, and by exciting them, agitate the solid parts of the refracting or reflecting body, and by agitating them, cause the body to grow warm or hot; that the vibrations thus excited are propagated in the refracting or reflecting medium or substance much after the manner that vibrations are propagated in the air for causing sound, and move faster than the rays, so as to overtake them; and that when any ray is in that part of the vibration which conspires with its motion, it easily breaks through a refracting surface, but when it is in the contrary part of the vibration which impedes its motion, it is easily reflected; and, by consequence, that every ray is successively disposed to be easily reflected or easily transmitted by every vibration which overtakes it.

CHAP. VIII.

OF THE PERMANENT COLOURS OF NATURAL BODIES, AND THE ANALOGY BETWEEN THEM AND THE COLOURS OF THIN TRANSPARENT PLATES.

A It has already been ſhewn (272, E), that the colours of natural bodies conſiſt in a diſpoſition to reflect the rays of one ſort of light more copiouſly than the reſt. But their conſtitution, whereby they reflect ſome rays more copiouſly than others, remains to be diſcloſed.

B Thoſe ſuperficies of tranſparent bodies reflect the greateſt quantity of light, which have the greateſt refracting power; that is, which intercede mediums that differ moſt in their refractive denſities. And in the confines of equally refracting mediums there is no reflection.

C The analogy between reflection and refraction will appear by conſidering that the moſt refractive mediums totally reflect the rays of light at leſs angles of incidence, as was before ſhewn (270, A). But the truth of the propoſition will further appear by obſerving, that in the common ſuperficies of two tranſparent mediums, the reflection is ſtronger or weaker, accordingly as the ſuperficies hath a greater or leſs refractive power. If any tranſparent ſolid be immerged in water, its

its reflection becomes much weaker than before, and ſtill weaker if immerged in a fluid whoſe refracting power is yet ſtronger than that of water. If water be diſtinguiſhed into two parts by an imaginary ſurface, the reflection in the confine of thoſe two parts is none at all. In the confine of water and ice it is very little; in that of water and oil ſomething greater; in that of water and ſal-gemm ſtill greater; and in that of water and glaſs, or cryſtal, or other denſer ſubſtances ſtill greater, accordingly as thoſe mediums differ more or leſs in their refractive powers. The reaſon then why uni- D form pellucid mediums, as water, glaſs or cryſtal, have no ſenſible reflection, but at their external ſuperficies, where they are adjacent to other mediums of a different denſity, is that all their contiguous parts have one and the ſame degree of denſity.

The leaſt parts of almoſt all natural bodies, are E in ſome meaſure tranſparent: and the opacity of bodies ariſes from the multitude of reflections cauſed in their internal parts.

This may be eaſily ſeen by viewing ſmall ſub- F ſtances with the microſcope or magnifying glaſs, for they appear for the moſt part tranſparent. And it may alſo be tried by means of the light received through a hole into a dark chamber. For any ſubſtance, how opake ſoever, if it be reduced to a ſufficient thinneſs, and applied to the hole, will appear manifeſtly tranſparent. Only white metalline bodies muſt be excepted, which, by reaſon of

their very great denſity, ſeem to reflect almoſt all the light incident on their firſt ſuperficies, unleſs by ſolution in menſtruums, they be reduced into very ſmall particles, and then they alſo become tranſparent.

G Between the parts of opake or coloured bodies are many ſpaces, either empty or repleniſhed with mediums of other denſities; as water between the tinging corpuſcles with which any liquor is impregnated, air between the aqueous globules that conſtitute clouds and miſts; and for the moſt part, ſpaces void both of air and water, but yet, perhaps, not void of all ſubſtance, between the parts of hard bodies.

A The truth of this is evinced by the two precedent propoſitions (F, G): for, by the ſecond, there are many reflections made by the internal parts of bodies, which would not happen if the parts of thoſe bodies were continued without any ſuch interſtices between them; becauſe reflections are only made in ſuperficies which intercede mediums of different denſities (293, D).

B A yet farther proof that the opacity of bodies ariſes from this diſcontinuation of their parts may be had, by conſidering that opake ſubſtances become tranſparent, by filling their pores with any ſubſtance of an equal or nearly equal denſity with their parts. Thus, paper dipped in water or oil, the oculus mundi ſtone ſteeped in water, linen cloth oiled or varniſhed, and many other ſubſtances ſoaked in ſuch liquors as will intimately pervade their

their pores, become by that means more tranſparent than otherwiſe; ſo, on the contrary, the moſt tranſparent ſubſtances may, by evacuating their pores, or ſeparating their parts, be rendered ſufficiently opake, as ſalts, or wet paper, or the oculus mundi ſtone, by being dried; horn, by being ſcraped; glaſs, by being reduced to powder, or otherwiſe flawed; turpentine, by being ſtirred about with water till they mix imperfectly; and water, by being formed into many ſmall bubbles, either alone in the form of froth, or by ſhaking it together with oil of turpentine, or ſome other convenient liquor with which it will not perfectly incorporate.

The parts of bodies and their interſtices muſt not be leſs than ſome definite bigneſs to render them opake and coloured. C

For the opakeſt bodies, if their parts be ſubtilely divided, as metals, by being diſſolved in acid menſtruums, &c. become perfectly tranſparent. And it may alſo be remembered, that the black ſpot near the point of contact of the two plates of glaſs being of ſome conſiderable breadth, tranſmitted the whole light where the glaſſes did not abſolutely touch (281, Y). And the reflection at the thinneſt part of the ſoap-bubble was ſo inſenſible as to make that part appear intenſely black, by the want of reflected light (284, I). D

On theſe grounds it is, that water, ſalt, glaſs, ſtones, and ſuch like ſubſtances, are tranſparent. For on many conſiderations they ſeem to be as full of pores or interſtices between their parts as other bodies E

are, but yet their parts and interſtices to be too ſmall to cauſe reflection in their common ſurfaces.

F The tranſparent parts of bodies, according to their ſeveral ſizes, muſt reflect rays of one colour, and tranſmit thoſe of another, on the ſame ground that thin plates or bubbles do reflect or tranſmit thoſe rays. And this appears to be the ground of all their colours.

G For if a thin body or plate, which, being of an even thickneſs, appears all over of one uniform colour, ſhould be ſlit into threads, or broken into fragments of the ſame thickneſs with the plate; there is no reaſon why every thread or fragment ſhould not keep its colour, and by conſequence, why a heap of thoſe threads or fragments ſhould not conſtitute a maſs or powder of the ſame colour which the plate exhibited before it was broken. And the parts of all bodies being like ſo many fragments of a plate, muſt on the ſame grounds exhibit the ſame colours.

H Now, that they do ſo, will appear by the affinity of their properties. The finely coloured feathers of ſome birds, and particularly thoſe of peacocks tails, do in the very ſame part of the feather appear of ſeveral colours in ſeveral poſitions of the eye. Likewiſe the fine-ſpun webs of ſome ſpiders appear coloured; and the fibres of ſome ſilks, by varying the poſition of the eye, do vary their colours. Alſo the colours of ſilks, cloths, and other ſubſtances which liquids can eaſily penetrate, become more faint by being wetted, much after the manner of the

plate

plate of Muſcovy glaſs, and recover their vigour again by being dried.

The air reflects the blue rays moſt plentifully, and muſt therefore tranſmit the red, orange, and yellow more copiouſly than the other rays. If the light of the ſetting-ſun, by paſſing through a long tract of air, be diveſted of the more reflexible rays, the green, blue indigo, and violet, the remainder, which is tranſmitted, will illuminate the weſtern clouds with an orange colour; and as the Sun ſets more and more, the tract of air through which the rays muſt paſs becomes longer, the yellow and orange are reflected, and the clouds grow more deeply red, till at length the diſappearance of the Sun leaves them of a leaden hue by the reflection of the blue light from the air. A ſimilar change of colour is obſerved on the ſnowy tops of the Alps in Switzerland, and the ſame may be ſeen, though leſs ſtrongly, on the eaſtern and weſtern fronts of white buildings; St. Paul's Church at London is a good object of this kind, and is often at ſun-ſet tinged with a conſiderable degree of redneſs. The ſame cauſe likewiſe occaſions the Moon in an eclipſe to aſſume a ruddy colour by the light tranſmitted through the atmoſphere (156, N, O). I

The parts of bodies, on which their colours depend, are denſer than the medium which pervades their interſtices. K

For if they were not, the variation of colour, ariſing from the various obliquities of the incident light, (283, E. 285 K) would compound a mixt and imperfect colour, and never ſo vivid as experience evinces. L

evinces. But when the parts are much denſer than the ambient medium, this variation is not ſo conſiderable; and therefore, the rays which are reflected leaſt obliquely may predominate over the reſt, ſo much as to cauſe a heap of ſuch particles to appear very intenſely of their colour.

M And hence the magnitude of the component parts of natural bodies may be conjectured by their colours.

N For, ſince the parts of theſe bodies are of about the ſame denſity as water or glaſs, as by many circumſtances is obvious to collect, it is highly probable that they exhibit the ſame colours with a plate of equal thickneſs. That colour being known, the thickneſs may be eaſily found by the preceding obſervations.

CHAP. IX.

OF THE INFLECTIONS OF THE RAYS OF LIGHT WHICH PASS IN THE VICINITIES OF BODIES.

O IT is obſervable, that if a beam of the Sun's light be let into a dark room through a very ſmall hole, the ſhadows of things in this light will be larger than they ought to be if the rays went on by the bodies in ſtrait lines, and that theſe ſhadows have three parallel fringes, bands, or ranks of colours adjacent to them. The principal circumſtances of the phenomenon are as follow:

P If a beam of the Sun's light be admitted into a darkened chamber through a hole of the breadth of

of the forty-second part of an inch, or thereabouts, the shadows of hairs, thread, straws, and other small bodies, appear considerably broader than they would be if the light passed by them in strait lines. For example; a hair, whose breadth was the 280th part of an inch, being held in this light at the distance of about twelve feet from the hole, did cast a shadow which, at the distance of four inches from the hair, was the sixtieth part of an inch broad, that is, above four times broader than the hair; and at the distance of ten feet, was the eighth part of an inch broad, that is, thirty-five times broader.

Nor is the effect altered by an alteration in the Q
density of the medium contiguous to the hair, for its shadow at like distances was equal, whether it was in the open air, or inclosed between two plates of wet glass, care being had that the incidence and emergence of the ray was perpendicular to the glasses. Scratches on the surface or veins in the body of polished glasses did also cast the like broad shadows. And therefore the great breadth of these shadows must proceed from some other cause than the usual refraction which might arise from any action of the ambient medium.

Let the circle X (fig. 84) represent the middle R
of the hair; ADG, BEH, CFI, three rays passing by one side of the hair at several distances; KNQ, LOR, MPS, three other rays passing by the other side of the hair at the like distances; D, E, F, and N, O, P, the places where the rays are bent in their passage by the hair; G, H, I, and Q, R, S,

the places where the rays fall on a paper, G Q; I S the breadth of the ſhadow of the hair caſt on the paper; and T I, V S, two rays which fall on the points I and S, without being at all deflected by the action of the hair. Then it is manifeſt, that all the rays between T I and V S are bent in paſſing by the hair, and turned aſide from the ſhadow I S, becauſe, if any part of the light were not bent it would fall within the ſhadow, and there illuminate the paper, contrary to experience. And becauſe, when the paper is at a great diſtance from the hair, the ſhadow is broad, and therefore the rays T I and V S are at a great diſtance from each other, it follows that the hair acts upon the rays of light at a conſiderable diſtance in their paſſing by it. But becauſe the ſhadow of the hair is much broader in proportion to the diſtance of the paper from the hair when the paper is nearer to the hair than when it is at a great diſtance from it, it is evident that the action is ſtronger on the rays which paſs by at leaſt diſtances, and grows weaker and weaker accordingly as the rays paſs by at diſtances greater and greater, as is repreſented in the ſcheme.

8 The ſhadows of all bodies in this light are bordered with three parallel fringes or bands of coloured light, of which that contiguous to the ſhadow is broadeſt and moſt luminous, and that moſt remote from it is narroweſt, and ſo faint as ſcarcely to be viſible. If the light be received very obliquely on paper, or any other ſmooth white body, the colours may be plainly diſtinguiſhed in this order, viz. the

firſt

firſt or innermoſt fringe is violet, and deep blue next the ſhadow, and then light blue, green and yellow in the middle, and red without. The ſecond fringe is almoſt contiguous to the firſt, and the third to the ſecond, and both are blue within, and yellow and red without, but their colours are very faint, eſpecially thoſe of the third. The colours therefore proceed in this order from the ſhadow, violet, indigo, pale blue, green, yellow, red; blue, yellow, red; pale blue, pale yellow, and red.

If a larger beam of the Sun's light be admitted T into a dark chamber, and part of it received on the blade of a ſharp knife, whoſe plane interſects the direction of the beam at right angles, while the other part is ſuffered to paſs by the edge of the knife, and received on a paper at the diſtance of about three feet; this laſt light will appear to ſhoot out or ſend forth two faint luminous ſtreams both ways into the ſhadow, ſomewhat like the tails of comets. Theſe ſtreams being very faint, are ſo much obſcured by the light of the principal direct rays, that it is neceſſary, in order to ſee them with any degree of diſtinctneſs, to let the direct rays paſs through a hole in the paper on to a piece of black cloth. The light of the ſtreams is then perceptible on the paper to the diſtance of ſix or eight inches from the Sun's direct light each way, and in all the progreſs from that direct light decreaſes gradually till it becomes inſenſible.

If two knife-blades, with ſtrait edges, be ſo U fixed or ſet in a frame, that they may both be

ſituated

ſituated in the ſame plane, their edges parallel, and facing each other, and one of the blades moveable towards or from the other by means of a ſcrew, ſo that their paralleliſm may be always preſerved, a beam of light may be ſuffered to paſs between their edges, and the appearances are the following: when the knives are at a conſiderable diſtance, ſo that the intromitted beam is broad, the ſtreams of light which ſhoot both ways into the ſhadow are ſcarce viſible, for the reaſon already mentioned, and the edges of the ſhadows are not bordered with coloured fringes, they becoming ſo broad that they run into each other, and by joining, form one continued light or whiteneſs at the beginning of the ſtreams. As the knives approach each other the fringes of colour appear on the confine of each ſhadow, becoming diſtincter and larger until they vaniſh, which happens when the edges are diſtant ſomewhat more than the 400th part of an inch. After the fringes have diſappeared, the line of light, which was in the middle between them, grows very broad, enlarging itſelf both ways into the ſtreams of light afore-mentioned; and when the knives are diſtant above the 400th part of an inch, the light parts in the middle, and leaves a ſhadow between the two parts. And as the knives ſtill approach each other, the ſhadow grows broader, and the ſtreams ſhorter at their inward ends, which are contiguous to the ſhadow, till upon the contact of the

knives

knives the whole light vaniſhes, leaving its place to the ſhadow *.

From theſe and ſome other experiments of the ſame tendency, it may be inferred,

That all bodies act upon the particles of light v attracting them when within a certain diſtance, and at greater diſtances repelling them; for the two comet-like ſtreams ſeem to be produced, the one by an attractive power exerted, by which the light is thrown into the ſhadow of the knife, and the other by a repulſion, by which it is turned towards the contrary part or region.

That theſe actions are ſtronger on thoſe rays w which paſs nearer the body than on thoſe which paſs at greater diſtances: conſequently thoſe rays which were parallel before their arrival in the vicinity of the body being variouſly deflected, muſt, after paſſing, diverge from each other; and, at the limit or diſtance at which attraction ceaſes, and repulſion begins, there muſt be a place at which the paſſing rays being very little affected by the action of the body, will proceed parallel, as before their arrival in its vicinity.

That this limitation or diſtance may differ in x rays of different colours, and cauſe the appearance

* The experiments of Newton on the inflection of light are few, and imperfect. Thoſe who have followed him in this delicate and highly important department of Natural Philoſophy, have done little more than add ſome inſulated facts to thoſe obſerved by him. The law followed by the powers that inflect light, and the limits of its action, are yet unknown.

of fringes: for, if the limit be leſs in the violet rays than in the red rays, the parallel rays of the violet colour will form a fringe, which ſhall be nearer the ſhadow of the body than that which is formed by the parallel rays of the red colour: and ſo of the intermediate colours will be formed intermediate fringes. It muſt, however, be confeſſed, that this ſuppoſition does not account for the repetition of the ſame colour at different diſtances.

CHAP. X.

OF THE POWERS BY WHICH BODIES REFLECT OR REFRACT THE RAYS OF LIGHT.

THE reflection of light is not cauſed by its impinging or ſtriking on the ſolid parts of bodies.

This will appear by the following conſiderations. Firſt, That in the paſſage of light out of glaſs into air, there is a reflection as ſtrong as in its paſſage out of air into glaſs, or rather a little ſtronger, and by many degrees ſtronger than in its paſſage out of glaſs into water. And it ſeems not probable, that air ſhould have more reflecting parts than water or glaſs. But if that ſhould poſſibly be ſuppoſed, yet it will avail nothing; for the reflection is as ſtrong, or ſtronger, when the air is drawn away from the glaſs, as when it is adjacent to it. Secondly, if light in its paſſage out of glaſs into air be incident more obliquely (270, A) than at an angle of 40 or

41 degrees, it is wholly reflected; if less obliquely, it is in a great measure transmitted. Now, it is not to be imagined that light, at one degree of obliquity, should meet with pores enough in the air to transmit the greater part of it, and at another degree of obliquity, should meet with nothing but parts to reflect it wholly; especially considering that in its passage out of air into glass, how oblique soever be its incidence, it finds pores enough in the glass to transmit the greatest part of it. If any one suppose that it is not reflected by the air, but by the outmost superficial parts of the glass, there is still the same difficulty: besides, that such a supposition is unintelligible, and will also appear to be false, by applying water behind some part of the glass instead of air. For so in a convenient obliquity of the rays, suppose of 45 or 46 degrees, at which they are all reflected where the air is adjacent to the glass, they shall be in great measure transmitted where the water is adjacent to it; which argues that their reflection depends on the constitution of the air and water behind the glass, and not in the striking of the rays upon the parts of the glass. Thirdly, If the colours made by a prism placed at the entrance of a beam of light into a darkened room be successively cast on a second prism (271, c) placed at a distance from the former, in such manner that they are all alike incident upon it, the second prism may be so inclined to the incident rays, that those which are of a blue colour shall be all reflected by it, and yet those of a red colour pretty

copiously transmitted. Now, if the reflection be caused by the parts of air or glass, it may be demanded why, at the same obliquity of incidence, the blue should wholly impinge on those parts, so as to be all reflected, and yet the red find pores enough to be in great measure transmitted. Fourthly, Where two glasses touch one another there is no sensible reflection (281, v), and yet no reason can be given why the rays should not impinge on the parts of the glass as much when contiguous to other glass as when contiguous to air. Fifthly, When the top of a soap water bubble, by the continual subsiding and exhaling of the water, becomes very thin, there is such a little and almost insensible quantity of light reflected from it, that it appears intensely black (284, 1); whereas, round about that black spot, where the water is thicker, the reflection is so strong as to make the water seem very white. Nor is it only at the least thickness of thin plates or bubbles, that there is no manifest reflection, but at many other thicknesses continually greater and greater. For we have seen that the rays of the same colour are by turns transmitted at one thickness, and reflected at another thickness for an indeterminate number of successions. And yet, in the superficies of the thin body, where it is of any one thickness, there are as many parts for the rays to impinge on as where it is of any other thickness. Sixthly, If reflection were caused by the parts of reflecting bodies, it would be impossible for thin plates or bubbles

bubbles at the ſame place to reflect the rays of one colour, and tranſmit thoſe of another. For it is not to be imagined, that at one place the rays which, for inſtance, exhibit a blue colour, ſhould accidentally ſtrike upon the parts, and thoſe which exhibit a red upon the pores, of the body; and then at another place, where the body is either a little thicker or a little thinner, that on the contrary, the blue ſhould hit upon its pores, and the red upon its parts. Laſtly, Were the rays of light reflected by impinging on the ſolid parts of bodies, their reflections from poliſhed bodies could not be ſo regular as they are. For in poliſhing glaſs with ſand, putty, or tripoly, it is not to be imagined that thoſe ſubſtances can, by grating and fretting the glaſs, bring all its leaſt particles to an accurate poliſh, ſo that all their ſurfaces ſhall be truly plane or truly ſpherical, and look all the ſame way, ſo as together to compoſe one even ſurface. The ſmaller the particles of thoſe ſubſtances are, the ſmaller will be the ſcratches by which they continually fret and wear away the glaſs until it be poliſhed; but be they ever ſo ſmall, they can wear away the glaſs no otherwiſe than by grating and ſcratching it, and breaking the protuberances, and therefore poliſh it no otherwiſe than by bringing its roughneſs to a very fine grain, ſo that the ſcratches and frettings of the ſurface become too ſmall to be viſible. And therefore, if light were reflected by impinging upon the ſolid parts of the glaſs, it would be ſcattered as much and as irregularly by the moſt poliſhed

glaſs as by the rougheſt. So that it remains a problem, how glaſs poliſhed by fretting ſubſtances can reflect light ſo regularly as it does. And this problem is ſcarce otherwiſe to be ſolved than by ſaying, that the reflection of a ray is effected not by a ſingle point of the reflecting body, but by ſome power of the body which is evenly diffuſed all over its ſurface, and by which it acts upon the ray without immediate contact: for that the parts of bodies do act upon light at a diſtance, has already been ſhewn (301, T, U).

D Now, if light be reflected, not by impinging on the ſolid parts of bodies, but by ſome other principle, it is probable that as many of its rays as impinge on the ſolid parts of bodies are not reflected, but ſtifled or loſt in the bodies. For otherwiſe, we muſt allow two ſorts of reflections. Should all the rays be reflected which impinge on the ſolid parts of clear water or cryſtal, thoſe ſubſtances would rather have a cloudy colour than a clear tranſparency. To make bodies look black in all poſitions, it is neceſſary that many rays be ſtopped, retained, and loſt in them; and it is difficult to conceive that any rays can be ſtopt and ſtifled in them which do not impinge on their parts.

E Bodies reflect and refract light by one and the ſame power, variouſly exercifed in various circumſtances.

This appears by ſeveral conſiderations. Firſt, Becauſe when light goes out of glaſs into air as obliquely

obliquely as it can poſſibly do, if its incidence be made ſtill more oblique, it becomes totally reflected (270, A). For the power of the glaſs, after it has refracted the light as obliquely as is poſſible, if the incidence be ſtill made more oblique, becomes too ſtrong to let any of its rays go through, and by conſequence cauſes total reflection. Secondly, Becauſe light is alternately reflected and tranſmitted by thin plates of glaſs for many ſucceſſions (285, L) accordingly as the thickneſs of the plate increaſes in an arithmetical progreſſion. For here the thickneſs of the glaſs determines whether that power by which glaſs acts upon light ſhall cauſe it to be reflected, or ſuffer it to be tranſmitted. And thirdly, Becauſe thoſe ſurfaces of tranſparent bodies which have the greateſt refracting power do alſo reflect the greateſt quantity of light (292, B, C).

The power by which bodies reflect and refract F
light, is the ſame as was ſhewn to be common to all bodies, and the cauſe of the inflection of the rays of light paſſing in their vicinities (303, U). For we muſt admit no more cauſes than are true, and ſufficient to explain the phenomena (6, I). Such a cauſe is this; its exiſtence being proved, and its adequacy to the explanation of the reflection and refraction of light eaſy to be ſhewn.

Let C D, (fig. 85) repreſent the ſurface of a G
tranſparent body A, contiguous either to a vacuum B, or other medium poſſeſſing a leſs power of reflecting or refracting the rays of light. Let E F repreſent an imaginary ſurface at ſuch a diſtance from

C D as to be situated at the limit of attraction, (303, W) that is to say, the space between E F and C D is that in which, if a ray of light pass, it will be attracted by the dense body A, and on the other side towards B, near the line E F, a ray of light will be repelled.

H Suppose now G H to be a ray of light passing within the rare medium B, obliquely towards the surface C D, and let the line or part K H denote its velocity. This motion may (23, T) be resolved into K I parallel, and I H perpendicular to C D. The attraction or repulsion exerted by the nearest parts of the body A (and the other parts may be neglected) or by those in the surface C D, must be assumed to act in the perpendicular to that surface, because no reason can be given why it should act towards one side more than another. It will therefore alter only the motion I H without affecting K I. When the light approaches E F it will be repelled; and if the force of repulsion in arriving at E F be greater than would generate the momentum I H, this last motion will be entirely destroyed before the light can arrive at the imaginary surface. The action of the repulsion, after it has destroyed I H, will, whatever may be its law, produce an equal velocity in the opposite direction. Consequently the ray will describe a motion compounded of H I and I L (equal to K I) and in the same direction; that is, it will pass through the line H L, making the angle of reflection I H L equal to the angle of incidence I H K (262, Y).

 Again,

Again, ſuppoſe M N to be a ray of light paſſing within the rare medium B, which either by the more direct courſe towards the ſurface or otherwiſe has the perpendicular part O N of its motion too great to be deſtroyed by the repulſion experienced in approaching E F. It will paſs that imaginary ſurface, ſuffering only a diminution of its velocity eſtimated in the perpendicular O N. While it goes forward towards C D, its velocity in the perpendicular will be continually augmented by the attractive force; and if the whole accelerating force exceed the whole retarding force, as in this caſe experience ſhews it does, the light will enter and proceed in the denſe body with a velocity in the perpendicular Q S, greater than it had before in O N; the parallel velocity P O or S R ſtill continuing the ſame. The ray Q T will for this reaſon make a leſs angle S Q R with the perpendicular than before, inſtead of continuing in the line N U; that is, it will be refracted towards the perpendicular by entering the denſe body (262, A). I

Again, ſuppoſe V W to be a ray of light paſſing within the denſe body A, obliquely towards the ſurface C D. Reſolve the motion repreſented by V W into V X and X W, the firſt parallel, and the latter perpendicular to C D. The ray will paſs out of the denſe body into the ſpace between E F and C D; where, if the force of attraction towards C D on a ray during its paſſage to E F be greater than the momentum X W in the contrary direction, this laſt motion will be entirely deſtroyed before the light K

can arrive at the imaginary ſurface. Whence it follows, that for reaſons ſimilar to thoſe uſed in ſpeaking of the ray G H (310, H), the ray V W will be again returned towards C D, with a velocity equal and contrary to X W, which, together with X Z, the continuation of the uniform and unaltered velocity Y X, will compound the actual motion W Z, making the angle of reflection X W Z equal to the angle of incidence X W Y (262, Y).

L Laſtly, ſuppoſe T Q to be a ray of light paſſing within the denſe body A, which either by the more direct courſe towards the ſurface, or otherwiſe, has the perpendicular part S Q of its motion too great to be deſtroyed by the attraction experienced in its paſſage to E F. It will paſs that imaginary ſurface, ſuffering only a diminution of its velocity eſtimated in the perpendicular S Q. When it has gone beyond E F, its velocity in the perpendicular will be continually augmented by the repulſive force; and if the whole accelerating force be leſs than the whole retarding force, as in this caſe experience ſhews it is, the light will enter, and proceed in, the rare medium with a velocity in the perpendicular N O, leſs than it had before in S Q; the parallel velocity R S or O P ſtill continuing the ſame. The ray N P will for this reaſon make a greater angle O N P with the perpendicular than before; that is, it will be refracted from the perpendicular by entering the rare medium (262, Z, A).

M From theſe conſiderations it is deduced alſo, that the rays of light are not refracted or reflected

all

all at once, but in refraction bent into a curve by the action of the body, ſo as to enter the ſurface of any medium more or leſs directly than they otherwiſe would have done, if its denſity had continued the ſame through the whole courſe of the rays. And in reflection, that the force acting in the direction of the perpendicular to the ſurface of a body, does not deſtroy the motion of the ray all at once, but bends it back in a curve. Which force, when it has deſtroyed that part of the motion of the ray which tended perpendicularly towards the common ſurface of the adjacent mediums, muſt reflect the ray with an equal angle and degree of velocity on the oppoſite ſide of the perpendicular to the point of incidence, or vertex of the curve. This is evident from what has al- N
ready been ſaid on the compoſition and reſolution of motion (23, T), and may, perhaps, without entering into particular explanations, be more readily conceived by attending to the motions of bodies projected obliquely from the Earth's ſurface; for here the aſcending or perpendicular part of the motion is gradually deſtroyed by the continually acting force, and a new, ſimilar, and equal motion is generated in the contrary direction, which, abſtracting the effect of the air's reſiſtance, cauſes the body to fall under an equal angle, and with the ſame velocity.

If the forces of bodies upon the particles of O
light be ſuppoſed to act equally after the ratio of the maſſes of the particles, the rays will be all

equally refracted or reflected, however different
P their maſſes, provided their velocities be equal.
If the ſame law of the forces be ſuppoſed, and the
velocities of the particles be various, thoſe which
move with leſs velocities will ſuffer a greater deflection than thoſe which move with greater velocities. The varying refrangibility and reflexibility of the rays of light muſt ariſe either from the various velocities of the particles themſelves, or from the action of bodies on the particles being ſtronger on ſome than on others, after the ratio of their
Q maſſes. If the various velocities were the cauſe, the moons of Jupiter, after being eclipſed, ought to appear illuminated with a variety of colours, in ſucceſſion, as the velocities of their conſtituent rays cauſed them reſpectively to arrive at the eye of the obſerver: and when light is diſperſed, by refraction, into its component colours, the quantity of this diſperſion ought in every medium to be equal at equal mean refractions of the whole ray: both which are contrary to experience.
R Whence it follows, in order to produce the variety of refraction or reflection which happens in the ſeveral rays of light, bodies muſt act on ſome of the particles of light more ſtrongly than upon others, after the ratio of their maſſes.

BOOK

BOOK II.

SECT. II.

Of Optics.

CHAP. I.

CONCERNING THE REFLECTION AND REFRACTION OF LIGHT BY SURFACES REGULARLY FORMED.

Before the diſcoveries of Sir Iſaac Newton A
had ſhewn the compoſition of white light, the ſcience of optics conſiſted of propoſitions in which the rays of light were always ſuppoſed to be equally refrangible or reflexible. And, indeed, though the diſperſion of light, when refracted into its component colours, is the greateſt obſtacle to the perfection of the inſtruments now made; yet on moſt occaſions, with reſpect to viſion, we may regard a ray of white light as ſtill continuing white, even after refraction. For the colours of the ſpectrum into which it is dilated, are ſo near each other, when the incidence is near the perpendicular, that to ſenſe they form a white very little differing from that of the incident ray. But in ſtrict- B

neſs,

neſs, the general principles of optics are true only of any ſingle kind of rays.

C That bodies are viſible only by means of the light which they emit or reflect, is too evident to need any particular proof; and that every point of an illuminated ſurface emits the rays of light in all directions, is clear from the viſibility of the ſurface, to an eye in any poſition whatſoever: for if any part or ſenſible point of the ſurface did not emit light in a ſuppoſed or given direction, that point, to an eye placed in that direction, muſt be inviſible. But this effect never happens.

D The rays which proceed from a point are neceſſarily divergent, but if they fall on a reflecting or refracting ſurface, they will be ſcattered in ſuch directions as the conſtruction of the ſurface pro-
E duces. If the ſurface be properly formed, the whole beam of rays may proceed, after reflection or refraction, either diverging from ſome other point, or parallel, or converging to a point.

F When the rays which are emitted or proceed from any point are conſidered, that point is called
G the radiant point; when the rays which proceed to any point are conſidered, that point is called the
H focus; and when the rays which proceed from a whole ſurface or object, are conſidered, the body of rays which is emitted from any one point, or as much of it as is applied to uſe, is called a pencil of rays.

I Since a pencil of rays emanating from any given point of ſpace, is the means by which the ſight

ſight aſſures us, that a body exiſts at or in that point, it is plain that we are liable to deception in that reſpect: for if the pencil be ſo affected, either by reflection or refraction, as to proceed with a different divergency or direction, that is, in the ſame manner as it would have proceeded if emitted from ſome other point of ſpace, the ſenſe will refer the place of the object to the point which is in the direction of the laſt courſe of the rays (263, c).

Thus, if M R (fig. 86) repreſent the ſection of K
a plane mirror, and O B an object, then the pencils O C and B D being reflected at C and D, will proceed to the eye at E, in the ſame manner as if emitted from points ſituated at I and M, and the ſame happening to the pencils which are emitted from the intermediate points between O and B, the ſenſe will refer the place of the object to I M. The ſame happens by refraction, as is clear from the conſideration of fig. 73. (263, E).

If a pencil of rays be rendered convergent, ſo L
as to meet and croſs each other in a point, they will afterwards diverge, and the ſenſe will refer the place of the radiant point or object to the focus of the convergent rays, from which the divergence was laſt made; and that rays of any ſort may be rendered thus convergent, either by reflection or refraction, is eaſily ſhewn.

Suppoſe R (fig. 87) to be a point, in any il- M
luminated or luminous object, which emits a pencil conſiſting of ſeven rays of light, R A, R B, R C,

R D, R E, R F, R G; let the ray R A be received on a ſpeculum, ſo placed as to reflect it through the point S: let another ſpeculum be adapted to receive and reflect R B alſo through S; and, in like manner, let the other rays be reflected through the ſame point; and the point S will become a radiant point, by means of the divergent rays, and will affect the ſenſe in the ſame manner as if the rays actually flowed from a body placed there. If the ſpeculums be ſuppoſed to touch each other, they will form a polygonal concavity. Suppoſe now the number of rays, inſtead of ſeven, to be infinite; then the adapted reflecting ſurface A G, inſtead of polygonal, muſt become curve, by reaſon of the infinite number of ſides. The ſame reaſoning may be applied to rays, which, inſtead of being emitted from a point, or diverging, fall on the reflecting ſurface, either converging to a point, or parallel to each other. It is therefore poſſible to conſtruct a ſuperficies that ſhall reflect into a focus the rays of light, which, either by converging or diverging, are directed either to or from any particular point.

N Upon the ſame principles may be conſtructed ſpeculums, which ſhall cauſe the rays, after reflection, to diverge from any given point behind the
O reflecting ſurface. Thoſe ſpeculums, which cauſe the rays to become more divergent muſt be convex, and thoſe which cauſe them to become more convergent muſt be concave, as may eaſily be imagined.

The

The celebrated Archimedes, at the ſiege of Sy- P
racuſe, is ſaid to have deſtroyed the ſhips of Mar-
cellus, by a machine compoſed of ſpeculums. Since a plane ſpeculum, in theory, reflects all the light which is incident upon it, under the ſame affections with which it was incident; the rays of the Sun, which, as coming from a vaſtly diſtant object, may be eſteemed parallel, will be reflected parallel to each other; and conſequently will heat and illuminate any ſubſtance on which they fall after reflection, in the ſame manner as if the Sun ſhone directly upon it. Two ſpeculums, which reflect the Sun's light on the ſame ſubſtance, will heat it twice as much as the Sun's direct light. Three will, in like circumſtances, heat it three times as much. And, by increaſing the number of ſpeculums, a prodigious degree of heat may be produced; more than ſufficient to conſume and de-
ſtroy any inflammable ſubſtance.

Though a plane ſpeculum in theory is ſuppoſed Q
to reflect all the light which falls upon it, yet in practice almoſt half the light is loſt, on account of the inaccuracy of the poliſh, and the want of perfect opacity in the ſubſtance of the mirror; on which accounts it happens that a conſiderable part of the light is ſcattered in all directions, and another part is abſorbed by the body. The R
indefatigable Buffon, in the year 1747, was the firſt of the moderns who conſtructed a burning machine of this kind. It conſiſted of 168 quick-
ſilvered glaſſes or ſpecula, each 8 inches long and 6 broad,

6 broad, so contrived, that the focal distance might be varied, and also the number of glasses, as occasions required. In the month of March, 1747, with 40 glasses he burnt a plank, at the distance of about 70 feet.

S If a body of rays, which either proceeds parallel, or, by converging or diverging, respects a given point, fall on the intercedent surface of two mediums of different refracting powers, the rays may be so refracted, if the surface be rightly formed, as to proceed parallel, or to converge to, or to diverge from, some other point.

T Let the polygonal surface A B C D E F G (fig. 88) represent the surface intercedent between two mediums, the rarer being situated on the side towards R, and the denser towards S; and let a pencil, composed of seven rays, R A, R B, R C, R D, R E, R F, R G, be incident, each ray on a different plane, as represented in the figure. Suppose the ray R A to be received on the surface at A, with an angle of incidence that corresponds to the angle of refraction which deflects the ray to the point S. And suppose the ray A B to be received less obliquely, or at a certain less angle of incidence; its angle of refraction will also be less, and it will proceed to S. And let a similar adjustment of the planes at C, D, &c. be supposed, and the other rays will be refracted to the same point. Or if S be supposed the radiant point, the mediums being as before,
U the focus will be at R. It is therefore plain, that rays proceeding out of a rare into a dense medium are

are rendered more convergent by a convex ſurface, and rays, proceeding out of a denſe into a rare medium, are rendered more convergent by a concave ſurface; and the contrary. Let the pencil conſiſt of an infinite number of rays, and the polygonal ſurface, adapted to refract it to a point, will, by reaſon of the infinite number of its ſides, become a curve. The ſame argument may be applied to rays that are either convergent or parallel at their incidence on the refracting ſurface. Conſequently, the intercedent ſurface of two mediums may be ſo formed as to refract into a focus, or render parallel, or divergent thoſe rays, which, at their incidence, are either parallel, or do, by converging or diverging, reſpect any particular point.

From the eſtabliſhed laws of reflection and refraction, it is not difficult to inveſtigate the nature of the curves, into which the before-mentioned ſurfaces ought to be formed. But as the errors that ariſe from the uſe of ſpherical ſurfaces are very ſmall, and may be remedied by other means, and the mechanical or practical conſtruction of the required curves is very difficult, the parts of optical inſtruments are commonly formed ſpherical.

CHAP. II.

OF DIOPTRICS; OR THE REGULAR REFRACTION OF LIGHT.

X GLASS being a medium denſer and more refracting than the air, is uſed to make the tranſparent parts of optical inſtruments which are conſtructed to act by the principle of refraction. A piece of glaſs properly figured for that purpoſe is called a lens, and is diſtinguiſhed by the nature of its ſurfaces: thus A (fig. 89) is a plano-convex, B a double convex, C a plano-concave, D a double concave, and E a convex-concave.

Y The two firſt lenſes, A and B, nearly reſemble each other in their properties; for they refract converging or parallel rays to a point or focus, and refract diverging rays, ſo as either to make them meet in a focus or proceed leſs divergent than be-
Z fore. If A B (fig. 71) repreſent a double convex lens, and R a radiant point, then the rays which fall on the lens will be refracted to F, if the lens
A be of the requiſite convexity. For the rays that fall on the convex ſurface A C D are rendered more convergent, and are made to converge ſtill more by falling on the concave ſurface A D B (320, U). The two following lenſes, C and D (fig. 89), are referred to one ſpecies, on account of the reſemblance of their properties; for they render the incident rays more divergent than before, and therefore

fore cauſe diverging or parallel rays to diverge from an imaginary or virtual focus, and refract converging rays, ſo as either to make them diverge from an imaginary focus, or proceed leſs convergent than before. If A B (fig. 91) repreſent a B double concave lens, and R a radiant point, then the rays which fall on the lens will be rendered more divergent, and will proceed as if they had proceeded from the point F, which is called the virtual focus. The fifth lens E reſembles A and C B, if its convexity be deeper, or a portion of a leſs ſphere than its concavity: but if the concavity be deepeſt, its properties reſemble thoſe of C and D.

In the four firſt lenſes, the changes made in the D courſe of the rays are more conſiderable the more the ſurfaces are curved; but in the laſt the changes are more conſiderable, the more the curvities of the two ſurfaces differ from each other.

A right line, as R F (fig. 90) paſſing through E the center of any lens, and perpendicular to both its ſurfaces, is called the axis of the lens. The focus of rays that reſpect the axis, either by falling parallel to it, or diverging from or converging to a point ſituated in it, is found in the axis, and is called the principal focus.

A right line drawn from the point of convergence F or divergence of any pencil of rays incident on a lens, through the center of the lens, will paſs through the focus of that pencil, if the point of

convergence or divergence be not ſituated far from the axis.

G The rays of light which diverge from the focus after paſſing through a lens, will occaſion the ſenſe to refer to that point, as if occupied by a lucid object (316, I); the focus, therefore, may be ſaid to be the picture or image of the radiant point. And as a ſurface may be conceived to be compoſed of an indefinite number of radiant points, the like number of focal points will appear, and conſequently a ſurface will be formed that will be
H the image of the radiant ſurface. Let O B (fig. 92) repreſent an object, and L N a double convex lens; from O and B through C the center, draw the lines O C I and B C M, and the foci of the points O and B will be found at I and M in thoſe lines (323, F), more or leſs diſtant from C, as the curvity of the ſurfaces of the glaſs is leſs or greater. The foci of the radiant points ſituated between O and B will be found between I and M, by the ſame proceſs. Conſequently an image will be there formed, reſembling the object, from each point of which rays of light will diverge in the ſame manner as from a real object; and its poſition, by reaſon that the rays croſs at C, will be inverted, or contrary to
I the object itſelf, as appears by the figure. And becauſe the triangles O C B and I C M are ſimilar, the linear magnitudes of the image and the object be to each other reſpectively as their diſtances from the lens; for,

As the ſide c o, or diſtance of the object from the lens,

Is to the ſide o b, or length of the object,

So is the ſide c i, or diſtance of the image,

To the ſide i m, or length of the image.

Again, let o b (fig. 93) repreſent an object, K
and l n a double concave lens; draw o c and b c,
and the virtual foci of the points o and b will be
found at i and m in thoſe lines (323, f) more or
leſs diſtant from c, as the curvity of the ſurfaces
of the glaſs is leſs or greater. The intermediate
points of the object will have their intermediate
foci between i and m, and the poſition of the image
will be erect as well as the object. And becauſe L
the triangles o c b and i c m are ſimilar, the linear
magnitudes of the object and image will be as
their diſtances from the lens.

Hence it may be eaſily conceived, how convex M
lenſes become burning-glaſſes. For as the object
and image, if viewed from the center of the lens
ſubtend the ſame angle, and the Sun is ſeen under
an angle of about half a degree, we may readily
find the denſity of the rays which form its image
in the focus of any lens. For example, if a lens, N
four inches broad, collect the Sun's rays into a
focus, at the diſtance of one foot, or twelve inches,
the image will not be more than $\frac{1}{10}$ of an inch
broad. The ſurface of this little circle, there-
fore, will be 1600 times leſs than the ſurface of
the lens, and conſequently the Sun's light muſt
be ſo many times denſer within that circle. No

wonder, then, that it burns with a degree of violence and ardor far exceeding that of any culinary fire.

If a paper or white ſubſtance be held in the focus of a convex lens, the ſeveral foci of the radiant points of objects ſituated on the other ſide of the lens will illuminate as many points on the paper; which illuminated points agreeing in relative ſituation, intenſity, and colour with thoſe of the objects themſelves, will depict an exact and lively perſpective view of the ſame, though by reaſon of the croſſing of the rays, it will be inverted. But this phenomenon is ſcarcely to be ſeen, if any light be permitted to fall on the paper beſides that which paſſes through the lens; for which purpoſe the lens may be fixed in the window-ſhutter of a darkened chamber, as we ſhall have occaſion to remark in future.

CHAP. III.

OF THE EYE; AND OF VISION.

If the conſtruction of the univerſe were not ſo P evident a proof of the exiſtence of a ſupremely wiſe and benevolent Creator, as to render particular arguments unneceſſary, (162, A) the ſtructure of the eye might be offered as one, by no means of the leaſt. This inſtance, among numberleſs others, demonſtrating that the beſt performances of art are infinitely ſhort of thoſe which are continually produced by the divine mechanic.

Though the apparatus, by which the eye is Q preſerved and kept in a ſtate proper for the quick motion and accurate direction towards the object to be viewed, is well worth attention and remark; yet, as it does not immediately come under our notice as illuſtrative of the principles of optics, we ſhall conſider only the globe of the eye, or organ by which viſion is performed.

The eye is compoſed of ſeveral tunics or inte- R guments, one within the other, and is filled within with tranſparent humors of different refractive denſities. The external tunic called the ſclerotica, is white on the anterior part, except a circular portion immediately in front, which is tranſparent, and more convex than the reſt of the eye: this tranſparent part is called the cornea. Immediately adherent to the ſclerotica within, is the choroides,

or uvea, which, at the circumference of the cornea, becomes the iris, being expanded over great part of its ſurface, though not contiguous to it. The iris is compoſed of two kinds of muſcular fibres; the one ſort tend like the radii of a circle towards its center, and the others form a number of concentric circles round the ſame center. The central part of the iris is perforated, and the orifice, which is called the pupil, is of no conſtant magnitude; for, when a very luminous object is viewed, the circular fibres of the iris contract, and diminiſh its orifice; and on the other hand, when objects are dark and obſcure, the radial fibres of the iris contract, and enlarge the pupil ſo as to admit a greater quantity of light into the eye. The iris is variouſly coloured in different perſons, but according to no certain rule: in general, they whoſe hair and complection are light coloured, have the iris blue or grey; and on the contrary, thoſe whoſe hair and complexion are dark, have the iris of a deep brown. But what ſpecific difference this may occaſion in the ſenſe, or whether any at all, is not diſcoverable. Within the uvea is another membrane, which at the circumference of the cornea becomes fibrous, and is called the ligamentum ciliare. This ligament is attached to the circumference of a double convex lens, whoſe axis correſponds with the center of the pupil; and which, by means of the fibres, can be altered in a ſmall degree in poſition, and perhaps in figure. The lens is termed the cryſtalline humor; and is included in a very ſtrong and

tranſ-

tranſparent membrane, called the arachnoides. Between the cryſtalline humor and the cornea is contained a clear tranſparent fluid, called the aqueous humor; and between the cryſtalline humor and the poſterior part or bottom of the eye is included another clear tranſparent fluid, which is termed the vitreous humor. The refractive denſity of the cryſtalline is greater than thoſe of the humors that ſurround it. On the ſide next to the noſe a nerve is inſerted in the bottom of each eye, about twenty-five degrees from the axis of the cryſtalline, which, after entering the eye, is ſpread into an exceeding fine coat of network, termed the retina. Laſtly; a very black mucus or ſlime is ſpread over all the internal parts of the eye, that are not tranſparent, except the anterior part of the iris, which, as before obſerved, is coloured.

In the figure, the three concentric circles A B C S (fig. 94) repreſent the coats of the eye. The external coat, or ſclerotica, is tranſparent, and more convex between A and B, A K B being the cornea. The ſecond tunic, or uvea is fibrous between D and I, and between G and H, and is there called the iris; the hole I H is the pupil. The third coat becomes fibrous between D and E, and between G and F, being there called the ligamentum ciliare, and is attached to the circumference of the lens or cryſtalline humour E F. The cavity or chamber A E F B is filled with the aqueous humor, and the chamber D N G F E is filled with the vitreous humor. At N is inſerted the optic nerve, the expanſion of which,

which, over the internal furface D N O, is the retina.

T The manner in which the eye acts upon the rays of light may be thus explained. Let O L reprefent an object, and fuppofe a pencil of light to proceed from O, and enter the eye; then, becaufe the cornea is a convex concave lens, whofe convexity is greateft, (323, c) the rays will be rendered more convergent in paffing through it; and if the cryftalline be properly formed, they will be refracted by it into a focus at C on the retina. The fame will happen to the pencil which proceeds from L, whofe focus will be M; and the foci of the intermediate points will be between M and C: confequently an inverted picture or image will be formed on the retina, and fenfation be produced by the action of the light on the expanfion of the optic nerve, which from thence is conveyed to the

U fenforium. And that the parts of the eye are adapted to produce fuch an image, appears likewife from experiment: for if the tunica fclerotica be carefully taken away from the back of the eye of any animal, the inverted picture of external objects may be feen on the thin membranes which remain. Neither is the inverfion of the image any obftacle to the mind's conceiving that the object is erect; for a focus at M may be confidered as the indication of the exiftence of a radiant point at L, and a focus at C may indicate the exiftence of a radiant point at O; and fo of others, the mind contem-

V plating the object itfelf, and not the image; befides which,

which, we have notions refpecting pofition that are not derived from the fight, whence we judge whether a wall is perpendicular or a plane level, &c. Thefe notions are derived from a perception of the direction in which gravity conftantly acts; to which direction we always refer. Whence it happens, that though the pofition of the eye be ever fo much changed, the idea of the pofition of objects in view remains unaltered. For example; if an obferver view an upright pole or ftaff, the image of the pole on the retina will be in a line at right angles to the opening of the eyelids, provided he holds his head upright; but if he vary the pofition of his head, the image will be formed in a different pofition, and upon a different part of the retina: notwithftanding which, he conftantly imagines the pole to be erect and unaltered.

Becaufe the foci of rays that differ in divergence are found at different diftances from the lens, thofe which diverge lefs coming to a focus fooner than thofe which diverge more, it is neceffary that the eye fhould be adapted fo as to act upon the rays that arrive from points at various diftances, and to bring them to a focus upon the retina. The natural ftructure of the eye is fuch, that parallel rays have their focus on the retina; and when the proximity of any object caufes its rays to fall with a greater divergency, the pupil of the eye contracts and excludes the moft divergent rays, at the fame time that the cryftalline is brought forward, and perhaps rendered more convex by means of the ligamentum

ligamentum ciliare, by which proviſions the focus
Y ſtill falls on the retina. This adjuſtment of the eye to the diſtances of objects gives the reaſon why we cannot view a near and a diſtant object at the ſame time; for, if a hair be held at a few inches diſtance between the eye and a remote object, ſuppoſe a tree at half a mile diſtance, the tree will appear confuſed and indiſtinct when the attention is fixed on the hair, and the ſame will be the caſe with the hair when the attention is fixed on the diſtant tree.

Z There are ſome eyes naturally ſo defective, that they cannot effect this adjuſtment. Thoſe which are replete with humors have the cornea and cryſtalline too convex, ſo that the pencils come to their foci before their arrival at the retina, where they fall in ſmall circular ſpaces inſtead of points, and by their interference render the image confuſed: on the other hand, if the humors be deficient in quantity, the cornea and cryſtalline are too flat, and the pencils of rays not being ſufficiently refracted, arrive at the retina before their union in their foci; whence ariſes the ſame confuſion in the image as in the former caſe. They whoſe eyes are imperfect in the firſt manner are called myopes, from their winking or cloſing their eyelids, but more commonly near-ſighted, becauſe they ſee very near objects diſtinctly, the divergency of the rays cauſing their foci to fall on the retina. They whoſe eyes are too flat are called preſbytæ, becauſe the imperfection of the ſight of old men being

being occaſioned by a decay of the humors, is generally of this kind. Both theſe imperfections may in a great meaſure be remedied by the uſe of proper ſpectacles. Since the rays converge too ſoon in the eyes of myopes, it is plain that a concave lens interpoſed between the object and the eye will cauſe the rays to fall more divergent, and conſequently will prevent their converging to a focus before their arrival at the retina. And the rays may be made to converge ſooner in the eyes of preſbytæ, by means of convex ſpectacles, ſo that they, being already convergent when they enter the eye, will be ſufficiently refracted by the cornea and cryſtalline to have their focus on the retina, and cauſe diſtinct viſion.

Theſe imperfections are much more frequently A
the conſequences of habit than is generally imagined. Studious men are generally near-ſighted, whereas ſailors, ſportſmen, and others, who are uſed to fix their attention on remote objects, are more ſubject to the contrary defect. The eyes of old men have another defect, namely, rigidity, or a want of the power of adjuſtment, ſo that it often happens that they require concaves for diſtant and convex lenſes for near objects, being capable only of ſeeing objects diſtinctly with the naked eye that are at a moderate diſtance. Every one ſhould avoid the uſe of ſpectacles as much as poſſible. For, though they render viſion more diſtinct, yet, they never fail to increaſe the defect of the eye, ſo as in time

to render it almoſt impoſſible to ſee without them with any degree of diſtinctneſs.

B The eyes of various animals are accommodated with great ſkill to the exigencies of their ſituations. In fiſhes the cornea is almoſt flat, that it may be no obſtacle to their ſpeed in the water, but this is compenſated by the cryſtalline, which is ſpherical, and therefore adapted to perform the whole neceſſary refraction of the rays. And in cats and ſome other animals that prey in the dark, the pupil of the eye is ſo variable as to admit more than an hundred times the quantity of light at one time than another. The human eye admits more than ten times the quantity of light at one time than at another, and perhaps the differences may be much greater in very dark places: it is not improbable but that the iris may be then almoſt entirely drawn back, and the pupil expanded to the whole ſurface of the cornea.

CHAP.

CHAP. IV.

OF REFRACTING MICROSCOPES; OR THE DIOPTRIC INSTRUMENTS, BY MEANS OF WHICH SMALL AND NEAR OBJECTS ARE MAGNIFIED.

THE apparent magnitude of any object is meaſured by the angle under which it is viewed by the eye; conſequently the apparent magnitudes of two or more objects may be the ſame, or may differ in any proportion, let their real magnitudes be what they will. Thus, the apparent magnitudes of C D, F G, and H I (fig. 95) are equal when viewed by the eye at E, becauſe they are ſeen under the ſame angle, though their real magnitudes are very different: and the apparent magnitude of A B is greater than thoſe of the former three, becauſe it ſubtends a greater angle, though its real magnitude is equal to that of C D, and leſs than thoſe of F G and H I. C

The image of any object on the retina will be greater or leſs in proportion to its apparent magnitude, and therefore the ſame object is ſeen more enlarged and diſtinct the nearer it is brought to the eye, provided its diſtance be ſufficiently great for the rays to fall nearly parallel on the pupil: at leſs diſtances it continues to be enlarged, but is confuſed. The leaſt diſtance is about ſix inches. The eye can juſt diſtinguiſh objects that ſubtend an angle of half a minute of a degree, in which D

caſe the image on the retina is leſs than the $\frac{1}{7200}$ part of an inch broad, and the object, ſuppoſing it ſix inches diſtant, about the $\frac{1}{1200}$ part of an inch broad. And all ſmaller objects are inviſible to the naked eye.

E The inſtruments by which thoſe ſmaller objects are rendered viſible are called microſcopes, and are conſtructed in two different methods. The one is, by the interpoſition of a convex lens, between the object and the eye, to render it diſtinct at a leſs diſtance than ſix inches, by which means its apparent magnitude increaſes as the diſtance is diminiſhed: and the other is, by placing the object ſo with reſpect to a convex lens that its focal image may be much greater than itſelf, and contemplating that image inſtead of the object. The firſt are called ſimple or ſingle microſcopes, and the latter compound or double.

Let E Y (fig. 96) repreſent the eye, and O B a ſmall object ſituated very near, ſo that the angle of its apparent magnitude O C B may be large. Then its image on the retina I M will alſo be large; but becauſe the pencils of rays are too divergent to be collected into their foci on the retina, it will be very confuſed and indiſtinct. Let the convex lens R S (fig. 97) be interpoſed, ſo that the diſtance between it and the object may be equal to the focal length at which parallel rays would unite, and the rays which diverge from the object and paſs through the lens will afterwards proceed, and conſequently enter the eye, parallel: they will therefore unite,

and

and form a diſtinct image on the retina, and the object will be clearly ſeen, though if removed to the diſtance of ſix inches, its ſmallneſs would render it inviſible. And ſince the apparent magnitudes G
of objects that ſubtend ſmall angles are nearly in the inverſe proportion of their diſtances, if the real magnitudes be equal, the proportion in which the object is magnified will be as ſix inches to its diſtance from the eye. Whence it follows, that the moſt convex lenſes, having the ſhorteſt focal diſtance of parallel rays, muſt magnify the moſt; for they permit the object to approach nearer the eye than thoſe do which are flatter. When the lens is not H
held cloſe to the eye, the object is amplified ſomewhat more; becauſe the pencils, which paſs at a diſtance from the center of the lens, are refracted inwards toward the axis, and conſequently ſeem to come from points more remote from the center of the object, as may be ſeen in fig. 98, where the pencils which are emitted from o and B, are refracted inwards, and ſeem to come from the points i and m.

A drop of water is a microſcope of this kind, by I
reaſon of its convex ſurface; for, if a ſmall hole be made in a plate of metal, or other thin ſubſtance, and carefully filled with a drop of water, ſmall objects may be ſeen through it very diſtinct, and much magnified. But there are ſome difficulties in the management of theſe, which ſmall glaſſes are free from, and therefore they are not much uſed. In fact, cheapneſs is their principal recommendation.

K The compound microſcope, by means of which the image is contemplated inſtead of the object, is of two kinds, the ſolar and the common double

L microſcope. The ſolar microſcope is thus conſtructed: let A C (fig. 99) repreſent the ſide of a darkened chamber, L N a convex lens, fixed oppoſite a perforation in A C, O B a ſmall object, and P Q a white ſcreen placed within the chamber oppoſite to the lens; then, if the object be placed at a due diſtance from the lens, the pencil of light which proceeds from the point O will converge to a focus on the ſcreen at I, and the pencil which proceeds from the point B will converge to a focus at M, and the intermediate points of the object will be depicted between I and M, forming a picture which will be as much larger than the object in proportion as the diſtance of the ſcreen exceeds that of the image from the lens (324, I). This is the principle on which the inſtrument acts, but it is uſual to add other auxiliary parts as a lens or ſpeculum to illuminate the object by converging the Sun's light upon it, &c. which cannot, with ſufficient brevity, be here enlarged upon. The ſolar microſcope is by far the moſt pleaſing in its effects, and leaſt offenſive to the eyes of any in uſe.

M In the common double microſcope the image is contemplated inſtead of the object, being viewed through a ſingle lens in the ſame manner as the object in a ſingle microſcope. Thus,

N Let L N (fig. 100) repreſent a double convex lens, and O B a ſmall object, ſo applied, that the

pencils

pencils of rays which emerge from it, and paſs through the lens, may converge to their reſpective foci, and form an inverted image at I M. This image will be as much larger than the object in proportion as its diſtance exceeds that of the object from the lens (324, I), and, if it be viewed through the lens F G, will again be magnified upon the principle of the ſingle microſcope (336, F) in proportion as its diſtance from the eye is leſs than ſix inches; the image formed by the firſt lens, which is called the object-glaſs, ſerving inſtead of an object for the ſecond, or eye-glaſs. But it is to be O
noted, that the image formed in the focus of a lens differs from the real object in a very eſſential particular; that is to ſay, the light being emitted from the object in every direction, renders it viſible to an eye placed in any poſition, but the points of the image formed by a lens or mirror emitting no more than a ſmall conical body of rays, which arrives from the glaſs, can be viſible only when the eye is ſituated within its confine. Thus, the P
pencil which is emitted from B in the object, and is made to converge by the lens to M, proceeds afterwards diverging towards H, and therefore never arrives at the lens F G, nor enters the eye at E. But the pencils that proceed from the points o and b will be received on the lens F G, and by it carried, parallel, to the eye; conſequently the correſpondent points of the image i and m will be viſible, and thoſe which are ſituate farther out towards I and M

Q will not be seen. This quantity of the image I M, or visible area, is called the field of view.

R Hence it appears, that if the image I M be large, a very small part of it will be visible, because the pencils of rays will for the most part fall without the eye-glass F G. And it is likewise plain, that a remedy which would cause the pencils, that proceed from the extremes O and B of the object, to arrive at the eye will render a greater part of it visible; or, in other words, enlarge the field of view.

S This is effected by the interposition of a broad lens D E (fig. 101) of a proper curvature at a small distance from the focal image. For, by that means the pencil B M, which would otherwise have proceeded towards H, is refracted to the eye, as delineated in the figure, and the mind conceives from thence existence of a radiant point at P, from which the rays last proceeded (316, I). In the same manner, the other extreme of the image is seen at Q, and the intermediate points are also rendered

T visible. On these considerations it is, that compound portable microscopes are usually made to consist of an object lens, L N, by which the image is formed, enlarged, and inverted, an amplifying lens, D E, by which the field of view is enlarged, and an eye-glass or lens, by means of which the eye is allowed to approach very near, and consequently to view the image under a very great angle of apparent magnitude*.

* The aberration of the refracted rays from the true focus, which arises from the spherical figure of the lens, D E, the pris-

The magic lanthorn is a microſcope upon the ſame principles as the ſolar microſcope, and may be uſed with good effect for magnifying ſmall tranſparent objects; but in general it is applied to the purpoſe of amuſement, by caſting the ſpecies or image of a ſmall tranſparent painting on glaſs upon a white wall or ſcreen, at the focal diſtance from the inſtrument. After what has already been ſaid, it will be eaſy to underſtand the following deſcription of its component parts.

In the inſide of a box or lanthorn is placed the candle or lamp C (fig. 102) whoſe light paſſes through the plano-convex lens N N, and ſtrongly illuminates the object O B, which is a tranſparent painting on glaſs, inverted and moveable before N N, by means of a ſliding piece in which the glaſs is ſet or fixed. This illumination is ſtill more increaſed by the reflection of light from a concave mirror, S S, placed at the other end of the box, that cauſes the light to fall upon the lens N N, as repreſented in the figure. Laſtly, a lens L L, fixed in a ſliding tube, is brought to the requiſite diſtance from the object O B, and a large erect image I M is formed upon the oppoſite wall.

matic colours that are ſeparated very much, and the loſs of light by reflection, which is moſt conſiderable when the refraction is greateſt, are the cauſes why in the beſt double microſcopes three or more lenſes are ſubſtituted inſtead of the ſingle amplifying lens, D E.

CHAP. V.

OF REFRACTING TELESCOPES; OR THE DIOPTRIC INSTRUMENTS, BY MEANS OF WHICH REMOTE OBJECTS ARE RENDERED LARGE AND DISTINCT TO THE VIEW.

w As the microſcope is calculated to obviate the defects of viſion with regard to objects, whoſe angles of apparent magnitude are too ſmall for ſight on account of the ſmallneſs of the objects themſelves, ſo teleſcopes are adapted to improve the ſenſe with reſpect to objects, whoſe angles of apparent magnitude are too ſmall for ſight by reaſon of their remoteneſs or diſtance. The intention of both inſtruments is the ſame, namely, to increaſe that angle, and, by conſequence, the teleſcope differs very little from the compound microſcope, except in ſome particulars of convenience.

x Let L N (fig. 103) repreſent a convex lens, and O B a diſtant object; then the pencils of rays will be collected into their reſpective foci, and form the inverted image I M, to which the eye, by means of the lens E F, may approach ſo near as to view it very large and diſtinct. This is the common aſtronomical teleſcope.

y But, as it is inconvenient to view objects on the earth inverted, there are uſually contrivances annexed to the teleſcope by which the image becomes erect as well as the object. The ſimpleſt of theſe

is

is the following, where a concave is ſubſtituted inſtead of the convex eye-glaſs.

Let L N (fig. 104) repreſent the object-glaſs as before, and O B a diſtant object. Then the pencils from the reſpective points of the object would converge to their foci, and form the inverted image I M, if the lens E E were not interpoſed. But the lens E E being a double concave, occaſions the rays to diverge more than before; ſo that the rays which are emitted from B in the object, inſtead of converging to M, are made to proceed parallel towards H. For the ſame reaſon the rays from O are made to proceed parallel towards K; the intermediate pencils being affected in the ſame manner. Now, ſince parallel rays cauſe diſtinct viſion, it is plain, that an eye placed in the pencil H, will conceive it to be emitted from ſome point, ſuppoſe m, ſituated in the laſt direction of the rays, and the image of B will be ſeen at m. By the ſame argument, the image of O will be ſeen at i, by an eye ſituated at K, and the like for the intermediate points. Therefore, an image will be ſeen at i m, erect or ſimilarly ſituated with the object itſelf.

This teleſcope repreſents objects very bright and clear, and as much magnified as the other does, but is unpleaſant in its uſe, on account of the contracted field of view. For the pencils, being rendered divergent with reſpect to each other, paſs moſtly on one or the other ſide, without entering the pupil of the eye, and therefore a very ſmall

part of the image can be seen at once: thus, if the eye be at H, it will view the point m, and if it be moved towards K, it will see in succession all the parts of the image towards i: but, as the pupil of the eye is not broad enough to receive the pencils H and K at the same time, the points m and i cannot be seen at once. The larger the pupil and the nearer it is placed to the eye-glass, the more pencils enter the eye at once. Consequently the field of view is largest under these circumstances, and in all other cases less.

B By the addition of two eye-glasses to the astronomical telescope, it is adapted to terrestrial objects, the field of view remaining the same. Thus, the lens F F (fig. 105) which is similar to E E, being placed at twice the focal distance for parallel rays from E E, receives the pencils of parallel rays after they have crossed each other at X, and forms an image at i m, similar and equal to I M, but contrary in position, or erect; which last image is viewed by the lens G G. This is the common telescope, and though, by reason of the number of lenses, it does not represent objects so bright as the foregoing, yet, its ample field of view makes it much more pleasing and useful *.

* The eye piece of telescopes is usually fitted up with five or more lenses, for reasons similar to those mentioned in the note, on page 340. Their distances are often adjusted in such a manner that they magnify the first image I M on the compound microscope principle.

The

The exhalations which continually arise from the Earth, render the air less transparent, (293) especially near the Earth, where the mixture is less complete, and therefore the celestial bodies are seen much more obscure when in the horizon than when at any considerable elevation; for in the first case, they are viewed through that part of the atmosphere which is contiguous to the surface of the Earth, and in the latter through a part which is at a greater distance. But this obscurity is the least part of the inconvenience. The rising exhalations have a kind of undulating motion, like that of smoke or steam, so that objects seen through them appear to have a tremulous or dancing motion, which is sensible even to the naked eye, if distant objects be viewed in a hot summer's day. Hence also the stars twinkle, and the shadow of lofty buildings have a tremulous motion. In telescopes this effect is still more perceptible, insomuch as to render them intirely useless, for terrestrial objects, when they augment the apparent magnitude more than eighty times*. But when objects in the heavens are viewed at any considerable altitude, instruments may be used which magnify many thousands of times. C D E

* That accurate and enlightened astronomer, Alexander Aubert, Esq. observes, that this undulation is the greatest when the telescope is not placed in the open air, but within a room. For the temperature of the room being seldom correspondent with that which obtains abroad, there is almost always a considerable undulation produced at the window where the streams of hot and cold air mix. Herschel uses his telescopes in the open air.

From

F From this want of tranſparency in the atmoſphere
ariſes that gradual diminution in the light of ob-
jects, which painters call the aerial perſpective; for,
if the air were perfectly tranſparent, an object
would be equally luminous at all diſtances, be-
cauſe the viſible area and the denſity of light de-
creaſe in the ſame proportion, namely, as the ſquare
G of the diſtance. It is from this gradual diminu-
tion of light, together with the angle of apparent
magnitude, that we eſtimate diſtances; and becauſe
the celeſtial bodies, when near the horizon, are
more obſcure, for the reaſon urged above (345, F),
though their reſpective apparent magnitudes remain
unaltered, or in a ſmall degree diminiſhed, we
adopt the notion of their being actually larger at
H that time. Thus, likewiſe, men ſeen through a
miſt appear gigantic, the obſcurity cauſing us to
imagine them more diſtant than they really are.
I But, in the caſe of the heavenly bodies, there is
another circumſtance that tends to deceive us in
our judgment of the diſtance: we conceive the ſky
to be a concave dome; and as the clouds towards
the horizon are evidently more diſtant than thoſe
near the zenith, we imagine the horizontal radius
K to be much longer than the vertical. From this
notion we regulate our ideas with regard to the diſ-
tance of the heavenly bodies, at different altitudes;
which diſtance, we ſuppoſe to be greater than they
are nearer the horizon, and we are conſequently
led to imagine, that they are larger at that time.

By

By the solar microscope and magic lanthorn, L
we have seen that the species of near objects may be cast on a screen in a darkened chamber. The camera obscura has the same relation to the telescope, as the solar microscope has to the common double microscope, and is thus constructed.

Let C D (fig. 106) represent a darkened cham- M
ber perforated at L, where a convex lens is fixed, the curvity of which is such, that the focus of parallel rays fall upon the opposite wall. Then, if A B be an object at such a distance, that the rays which proceed from any given point of its surface to the lens L, may be esteemed parallel, an inverted picture will be formed on the opposite wall. For the pencil which proceeds from A will converge to a, and the pencil which proceeds from B, will converge to b, and the intermediate points of the object will be depicted between a and b. This is one of the most pleasing and delightful experiments in optics, and never fails to strike the beholder with surprise and admiration. Its only defect is the inverted position of the picture, which may be remedied by several methods. But as they all tend to make the image less lively, they are seldom used.

CHAP. VI.

OF THE IMPERFECTIONS OF TELESCOPES, AND THEIR REMEDIES; AND OF THE ACHROMATIC TELESCOPE.

N SINCE the conſtruction of a teleſcope conſiſts in nothing more than viewing, by means of a microſcope or eye-glaſs, the image which is formed in the focus of the object-glaſs; it may ſeem eaſy to make a teleſcope with a given object-glaſs, that ſhall magnify in any aſſignable degree. For, if the eye-glaſs be rendered more and more convex, the eye may be permitted to approach nearer and nearer to the image, and conſequently to view it under an angle of apparent magnitude that ſhall be greater and greater, as required. But this is
O unattainable on two ſeveral accounts. The firſt is, that ſpherical ſurfaces do not refract the rays of light accurately to a point, as has already been obſerved; and the ſecond and moſt con-
P ſequential is, that the rays of compounded light, being differently refrangible, come to their reſpective foci at different diſtances from the glaſs, the more refrangible rays converging ſooner than thoſe which are leſs refrangible. This is evident from what has already been ſaid on that ſubject,
Q and is likewiſe confirmed by experiment; for a paper, painted intenſely red, and properly illuminated,

nated, will caſt its ſpecies, by means of a lens, on a ſcreen at a greater diſtance than will another blue paper by the ſame lens in like circumſtances. And here it may be noted, that the lens proper for this experiment muſt be very flat, or a portion of the ſurface of a large ſphere. Hence the ſpecies or image of a white object may be ſaid to conſiſt of an indefinite number of co- R loured images, the violet being neareſt, and the red fartheſt from the lens, and the images of intermediate colours at intermediate diſtances. The aggregate, or image itſelf, muſt therefore be in ſome degree confuſed, and this confuſion, being very much increaſed by the magnifying power, or eye-glaſs, renders it neceſſary to uſe an eye-glaſs of a certain limited convexity to a given object-glaſs. For which reaſon, if it be required to con- S ſtruct a teleſcope that ſhall magnify objects in a greater degree than a given teleſcope, the object-glaſs muſt be leſs convex, and of conſequence its focal diſtance longer. Thus an object-glaſs of 4 T feet focal length will bear an eye-glaſs of about 1 $\frac{1}{5}$ inch focus, and will magnify objects in length or diameter 40 times: one of 25 feet focal length will bear an eye-glaſs of 3 inches focus, and magnifies 100 times; and one of 100 feet will bear an eye-glaſs of ſix inches, and magnifies 200 times. It is alſo neceſſary to limit the aperture of the ob- U ject-glaſs, to exclude thoſe rays which are incident at too great diſtances from the center; for thoſe, being more refracted, are more particularly ſub-

ject to the irregularities which arife, either from
the figure of the glafs or the unequal refraction of
light. The diameter of the apertures of object
lenfes, of equal goodnefs, fhould be as the fquare
roots of their focal lengths.

v The great inconvenience and difficulty of ma-
naging the longer telefcopes, occafioned the phi-
lofophic world to fix their thoughts upon the
means of converging the rays of light without fe-
parating them into their component colours. The
expedients for that purpofe were firft perfected by
w Sir Ifaac Newton and Mr. Dollond. The focal
image in the telefcope of Sir Ifaac Newton is formed
by reflection from fpeculums or mirrors, and be-
ing therefore free from the irregular convergency
of the various rays of light, will admit of a much
larger aperture, and bear the application of a very
x great magnifying power. The difficulties which
attend this inftrument, are the tarnifhing of the
metalline fpeculums, and the very great accuracy
required in giving them the true figure, for an
error in a reflecting furface affects the direction
of the rays much more than a like error in a re-
y fracting furface. Yet this telefcope is, notwith-
z ftanding, the beft in ufe. Mr. Dollond's inven-
tion confifts in the ufe of a compound object-glafs,
which is ufually termed achromatic, or colour-
lefs, from its property; and the principal imper-
fection in the practice, is the difficulty of pro-
curing glafs that fhall be uniformly of the fame
refractive denfity. As we are now fpeaking of
dioptrics,

dioptrics, it will be more regular to deſcribe the achromatic teleſcope firſt, and refer the other to its place, where we ſhall explain the properties of inſtruments that act on the principle of reflection.

Becauſe the component rays of light differ A
from each other in refrangibility, they are ſeparated from each other by refraction, and becauſe they are all refracted ſo as to preſerve a conſtant ratio between the ſines of the angles of incidence and refraction, that ſeparation muſt be greateſt when the whole beam of light is moſt deflected from its courſe. From hence opticians have concluded, and there is a paſſage in Sir Iſaac Newton's * optics, that ſeems to confirm the opinion, that priſms, which deflect the whole beam of light equally out of its courſe at like incidences, will, however different their refractive denſities, occaſion alſo an equal ſeparation or divergency of the component rays: or in other words, that if the emergent refracted light from the ſurface of a given priſm be immediately received on the ſurface of a ſecond priſm, which ſhall refract it equally in the contrary direction, ſo that at its emergence, it ſhall proceed parallel to the firſt incident beam, this laſt emergent light will continue white, however different the matter of the ſecond priſm may be from that of the firſt. But this Mr. Dol- B
lond has ſhewn to be ill-founded, for, by his experiments it appears, that the different kinds of

* Book I. Part 2. Experiment VIII.

glaſs differ extremely with reſpect to the divergency of colours produced by equal refractions. He found that two priſms, one of white flint-glaſs, whoſe refracting angle was about 25 degrees, and another of crown-glaſs, whoſe refracting angle was about 29 degrees, refracted the beam of light nearly alike, but that the divergency of colour in the white-flint was conſiderably more than in the crown-glaſs; ſo that, when they were applied together, to refract contrary ways, and a beam of light tranſmitted through them, though the emergent continued parallel to the incident part, it was, notwithſtanding, ſeparated into component colours. Whence he inferred, that, in order to render the emergent beam white, it is neceſſary that the refracting angle of the priſm of crown-glaſs ſhould be increaſed; and by repeated experiments, he diſcovered the exact quantity. But this colourleſs emergent light was not then, by reaſon of the increaſed angle of the priſm of crown-glaſs, parallel to the incident ray, but was refracted towards the baſe of the laſt mentioned priſm.

C By theſe means he obtained a theory, in which refraction was performed without any ſeparation or divergency of colour, and which it was not difficult to apply in the conſtruction of the object-
D glaſſes of teleſcopes. Let A B E D (fig. 107) repreſent a double concave lens of white flint-glaſs, and A G D F a double concave of crown-glaſs; then the parts of the lenſes which are on the ſame ſide of the common axis, namely, A C B and A F G, may be

be conceived to act like two priſms, which refract contrary ways, and if the exceſs of refraction in the crown-glaſs A F G be ſuch as preciſely to deſtroy the divergency of colour cauſed by the flint-glaſs A C B, the incident ray S H will be refracted to X, without any production of colour. The ſame is alſo true of the ray s h, and of all the other incident rays, and conſequently the whole focal image formed by this compound object-glaſs will be achromatic, or free from colour which might ariſe from refraction. It will therefore bear a larger aperture, and greater magnifying power, and of courſe enlarge objects much more than a common refracting teleſcope of the ſame length.

It is more convenient on ſeveral accounts to 1
combine three lenſes together, one double concave of flint-glaſs between two convexes of different kinds of crown-glaſs; and the glaſſes may be ſo adjuſted to each other, as not only to form a focal image without the priſmatic colours, but alſo free from the defects which in other lenſes ariſe from their ſpherical figure.

The greateſt impediment to the conſtruction of large achromatic teleſcopes, as has been obſerved, (350, z) is the want of a flint-glaſs of an uniform refracting denſity. Fortunately for Dollond, this kind of glaſs was procurable, when he began to make achromatic teleſcopes, though the attempts

of many ingenious chemists have since been exerted to make it, without much success*.

CHAP. VII.

OF CATOPTRICS, OR THE REGULAR REFLECTION OF LIGHT; AND OF THE REFLECTING TELESCOPE.

0 It has been shewn, (317, M) that a surface may be constructed that shall reflect the rays of a given pencil of light, so as to make them either converge to a point, diverge from a point, or proceed parallel to each other. This surface may be either plane or curved.

51 A plane mirror reflects a pencil of light under the same circumstances as it was incident; that is to say, if a pencil, which emanates from a given point, be incident on the mirror, it is reflected so, that its rays proceed with the same divergency from another point, whose distance behind the mirror is equal to the distance of the radiant point

* The author has been informed, that the glass employed by Dollond in the fabrication of his best telescopes, was all of the same melting, or made at one time; and that, excepting this particular treasure, casually obtained, good dense glass for achromatic purposes was always as difficult to be procured as it is now.

8 before

before the mirror from the place of incidence: and if the pencils of rays, which emanate from a given surface, be incident on the mirror, they will be reflected so as to preserve the same inclinations to each other as before, and therefore will appear to proceed from a surface, whose magnitude and distance behind the mirror are exactly equal to those of the radiant surface (317, K). Hence it is, that plane mirrors reflect the species of objects, which are equal, like, and similar in position with the objects themselves.

Concave mirrors render the pencils of rays, I
which are incident upon them, more convergent, and convex mirrors render them more divergent, (318, O). If the mirrors be regularly formed, according to the proper curve, the convergent or divergent light of any pencil after reflection will respect some particular point of space.

A portion of a sphere, whose breadth is about K
fifteen degrees, differs very little from the curve surface, by which parallel rays would be made by reflection to converge to, or diverge from, a point. It is therefore in many cases used for that purpose, as being much easier to construct.

There is a great resemblance between the pro- L
perties of convex lenses and concave mirrors, and between the properties of concave lenses and convex mirrors. Convex lenses and concave mirrors M
form an inverted focal image of any remote object by the convergence of the pencils of rays: concave N
lenses and convex mirrors do, in general, form an

erect image in the virtual focus, by the diver-
O gence of the pencils of rays. In thoſe inſtruments, whoſe performances are the effects of reflection, the concave mirror is ſubſtituted in the place of the convex lens, and the convex mirror
P may be uſed inſtead of the concave lens: but their diſpoſitions with reſpect to each other, when combined, muſt neceſſarily differ from thoſe of lenſes, on account of the opacity of the one, and the tranſparency of the other.

Q Let A F (fig. 108) repreſent the poliſhed ſpherical ſurface of a concave mirror, and O B an object ſituated without the center of the mirror; then the pencil of rays, which is emitted from the point O, will fall on the mirror; and after reflection, converge to the focus I; the pencil from B will converge to M, and the like will happen to thoſe emitted from the intermediate points, whoſe foci will be found between I and M. There will conſequently be formed before the mirror an inverted focal image, reſembling that which is formed by a convex lens (324, H).

R Let A R (fig. 109) repreſent the poliſhed ſpherical ſurface of a convex mirror, and O B an object: then the pencil of rays, which is emitted from the point O, will fall on the mirror, and after reflection diverge from the virtual focus I; the pencil from B will emerge from M, and the like will happen to thoſe emitted from the intermediate points, whoſe virtual foci will be found between I and M. There will conſequently be formed

formed behind the mirror an erect focal image, reſembling that which is formed by a concave lens.

Let A R (fig. 110) repreſent a concave mirror, s whoſe center is C, and O B an object ſituated without the center. Through the center C, from O, draw the line O N, which will be perpendicular to the mirror at N, and will therefore repreſent both the incident and reflected ray, which proceeds from O, and is reflected at N: the focal repreſentation or image of O will conſequently be found in that line. Through C, from B, draw the line B V, and by the ſame reaſoning the focal image of B will be found in that line. Draw the line or ray O V, and it will be reflected ſo as to croſs the ray O N at I, the angle of reflection I V C being equal to the angle of incidence O V C. This interſection of the rays determines the focal point of O, which is I. From B to N draw the ray B N, and its reflection will determine the focus of B, which is M, and the image will be inverted.

Let A R (fig. 111) repreſent a convex mirror, T and the other repreſentations and conſtruction be as in the laſt figure. The focal repreſentations of O and B will be found in the lines O C and B C, and the reflected part of the ray O V will virtually croſs the line O C at I; the reflected part of the ray B N will alſo virtually croſs the line B C at M. Theſe interſections will determine the place of the focal image I M, which will be erect.

u Hence it appears to be the property of theſe mirrors, that the object and the image, if viewed from the center of the ſphere, are ſeen under equal angles; for, the angle O C B is equal to

v the angle I C M; and that the object and image, if viewed from the point of reflection, are ſeen under equal angles; for, the angle O V B is equal to the angle I V M. From this it is eaſy to find the poſition and magnitude of the focal image, if the poſition and magnitude of the object, and the diameter of the ſphere, of which the mirror is a part, be known.

w The reflecting teleſcope, which was made by Sir Iſaac Newton, was of the following form.

Let D E F G (fig. 112) repreſent a tube; at one end of which is placed the concave mirror A R, and let O B repreſent a diſtant object; then the pencils, which are emitted from the ſeveral points of its ſurface, will be collected, and form an inverted image i m. But by the interpoſition of the plane mirror K C, the rays are reflected, and the image is formed at I M, which is ſeen very much magnified by means of the plano-convex lens at L.

x The immenſely powerful teleſcopes of Herſchel are on this conſtruction. This capital artiſt, and moſt aſſiduous aſtronomer, has made ſeveral ſpeculums, which are ſo perfect as to bear a magnifying power of more than ſix thouſand times in diameter on ſome of the fixed ſtars*. The largeſt

* Phil. Tranſ. 1784.

teleſcope

teleſcope completed by him, and far exceeding in all reſpects any yet attempted, has an object ſpeculum of forty feet focal length.

The reflecting teleſcope, which is moſt in uſe at preſent, is compoſed of two concave mirrors of different radii. The larger concave A R (fig. 113) forms the focal image I m, which ſerves as an object for the ſmall mirror K C: a ſecond image i m is formed by the mirror, the rays paſſing through the amplifying lens L, which is placed in a hole or perforation in the center of the great mirror A R. This image is erect, and is viewed much enlarged through the eye-glaſs or lens P. Y

In good reflecting teleſcopes the object ſpeculum is not of a ſpherical form. Z

Reflecting microſcopes are ſometimes made; the method of conſtructing which, as alſo of other inſtruments, may be deduced from what has been ſaid concerning reflecting teleſcopes. A

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